Accuracy and Precision of Approximate Numbers
(MAT 131)

The result of mathematical operations done with approximate numbers is also an approximate number and must be rounded according to either the accuracy or the precision of the original numbers. The output cannot be more accurate or precise than the input.

Rounding off a number: When rounding a number to a certain place (e.g., the ones or the tens or the tenths place), discard all the digits to the right of that place. If the digit just to the right of that place is 5 or higher, round up. If it is less than 5, round down.

Round 2054 to the hundreds place	→	2100	(round up because of the 5)
Round 2054 to the tens place	→	2050	(round down because of the 4)
Round 2.974 to the tenths place	→	3.0	(round up because of the 7)

Accuracy: the number of significant digits

- non-zero numbers are significant
  541 has three significant digits; 2000 has one significant digit
- zeros used only to locate the decimal point are not significant
  0.0541 has three significant digits; 0.0002 has one significant digit
- zeros not needed just to locate the decimal point are significant
  2001 has four significant digits
  20.01 has four significant digits
  541.00 has five significant digits (the zeros are not necessary to locate the decimal point, so would not be written unless significant)

Operations: Multiplication, division, roots

Round the result to the accuracy of the least accurate number

$$\frac{(23.341)(0.03333)}{9.396} = 0.08280 \quad \text{(4 significant digits)}$$

$$\sqrt{0.003} = .05 \quad \text{(1 significant digit)}$$

Precision: the decimal position of the last significant digit

Operations: Addition and subtraction

Round the result to the precision of the least precise number

$83.5 + 138.34 + 72 = 294$
$13.62 + 3.2354 - 1.243 = 15.61$

Exact numbers

A number is exact if:

1) it is determined by definition. For example, by definition, 5,280 ft = 1 mile, so the 5,280 and the 1 are exact numbers. 60 min = 1 hour, and the 60 and the 1 are exact.

2) it is based on counting discrete units. For example, a number of people or a price in dollars would be an exact number.

In an exact number, all digits are significant. If prices are given in whole dollars, the price $1200 has 4 significant digits (otherwise, the number 1200 would have 2 significant digits).

Combined operations

1) When exact and approximate numbers are both involved in a calculation, the rounding of the result is determined only by the approximate number.

Given that 300 is exact:	300 + 4.6 = 304.6	(1 decimal place)
	300 * 4.6 = 1400	(2 significant digits)

2) When different operations are combined, the last operation determines how the result is rounded off.

$\dfrac{26.106 * 0.02106}{10.313 - 8.051} = 0.2431$ division done last; least accurate number has 4 significant digits

$0.035 - \dfrac{0.0456}{1.53} = 0.01$ subtraction done last; least precise number has 2 decimal places

When working problems on a calculator, do the entire calculation without rounding, then round appropriately at the end.