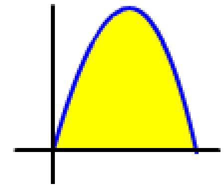
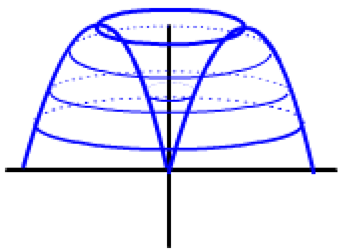
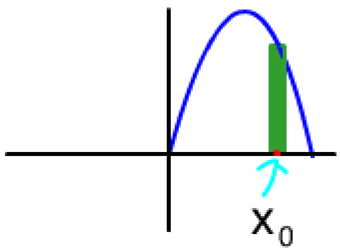
# SOLIDS OF REVOLUTION
# THE SHELL METHOD



Consider the area bounded by the graph of the function $f(x) = x - x^2$ and the x-axis.

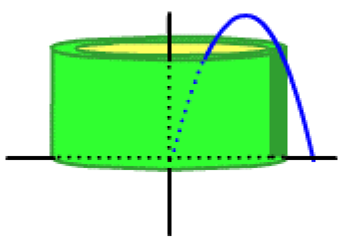


We want to calculate the volume of the solid obtained by rotating this area about the y-axis.

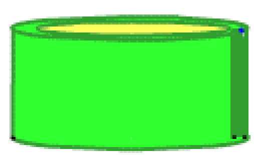



Choose some $x_0$ between 0 and 1. Draw a rectangle with height $f(x_0)$ and with very small width $\Delta x$.

Rotate this rectangle about the y-axis.



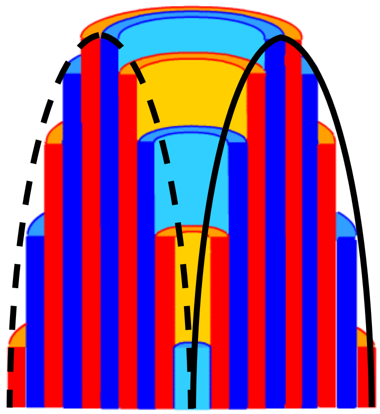
The result is a cylinder with a "very small side" like the side of a can:



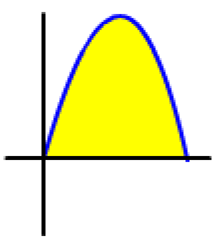
We call the cylinder a shell. It has radius $x_0$, height $f(x_0)$ and thickness $\Delta x$. We obtain the volume of the shell by multiplying its circumference by its height by its thickness:

$V = 2\pi r\, f(x_0)\, \Delta x$, or since the radius is $x_0$, $V = 2\pi x_0\, f(x_0)\, \Delta x$.

Now we fill the area with rectangles and rotate each of these rectangles about the y-axis to obtain cylinders with approximate volume $2\pi x_i\, f(x_i)\, \Delta x$.



The sum $\sum_{i=1}^{n} 2\pi x_i\, f(x_i)\, \Delta x$ is a Riemann sum for the integral $2\pi \int_0^1 x\, f(x)\, dx$.



The volume of the solid obtained by rotating this area about the y-axis is:

$$2\pi \int_0^1 x\, f(x)\, dx = 2\pi \int_0^1 x\,(x - x^2)\, dx$$

$$= 2\pi \int_0^1 (x^2 - x^3)\,dx$$

$$= 2\pi \left[ \frac{x^3}{3} - \frac{x^4}{4} \right]_0^1$$

$$= 2\pi \left[ \frac{1^3}{3} - \frac{1^4}{4} \right] - 2\pi \left[ \frac{0^3}{3} - \frac{0^4}{4} \right] = \frac{\pi}{6}$$