Partial Fractions – Cover-Up Method

The "cover-up" method is a quick way to determine the coefficients in a partial fraction decomposition when the denominator of a rational function has different (non-repeated) linear factors. The concept is as follows:

$$\frac{x^2+6x+9}{(x+1)(x-2)(x-4)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x-4}$$

Multiply each side by (x + 1):

$$\frac{x^2+6x+9}{(x-2)(x-4)} = A + \frac{B(x+1)}{x-2} + \frac{C(x+1)}{x-4}$$

Set x = - 1:

$$\left.\frac{x^2+6x+9}{(x-2)(x-4)}\right|_{x=-1} = A + 0 + 0 \qquad \rightarrow \qquad A = \frac{4}{15}$$

Multiply each side by (x – 2):

$$\frac{x^2+6x+9}{(x+1)(x-4)} = \frac{A(x-2)}{x+1} + B + \frac{C(x-2)}{x-4}$$

Set x = 2:

$$\left.\frac{x^2+6x+9}{(x+1)(x-4)}\right|_{x=2} = 0 + B + 0 \qquad \rightarrow \qquad B = -\frac{25}{6}$$

Multiply each side by (x – 4):

$$\frac{x^2+6x+9}{(x+1)(x-2)} = \frac{A(x-4)}{x+1} + \frac{B(x-4)}{x-2} + C$$

Set x = 4:

$$\left.\frac{x^2+6x+9}{(x+1)(x-2)}\right|_{x=4} = 0 + 0 + C \qquad \rightarrow \qquad C = \frac{49}{10}$$

The "cover-up" way of looking at it is to cover up the factor in the left-hand denominator that corresponds to the coefficient on the right, and evaluate the left side at the x value that makes that factor zero.

$$\left.\frac{x^2+6x+9}{\boxed{(x+1)}(x-2)(x-4)}\right|_{x=-1} = A = \frac{4}{15}$$

cover up

$$\left.\frac{x^2+6x+9}{(x+1)\boxed{(x-2)}(x-4)}\right|_{x=2} = B = -\frac{25}{6}$$

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$$\left.\frac{x^2+6x+9}{(x+1)(x-2)\boxed{(x-4)}}\right|_{x=4} = C = \frac{49}{10}$$

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