FUNCTIONS OF SEVERAL VARIABLES

A function of several variables is usually noted as f(x,y) or z = f(x,y).

For example f(x,y) = $x^2 + y^2$, or f(x,y) = $\frac{xy}{x^2 + y^2}$. x and y are the independent variables and z is the dependent variable. As a function, f(x,y) has a domain and a range. Just as we had with f(x) there are three restrictions which result in exclusions in the domain of real numbers:

1. Cannot divide by zero,
2. Cannot take the square root of a negative number,
3. There are no log's or ln's of a negative or zero number.

With the two independent variables, x and y, the domain will generally be all or part of the xy-plane, sometimes just in certain quadrants.

Suppose f(x,y) = ln(xy). Then, since xy cannot be negative or equal to zero, xy must be greater than zero, that is, xy > 0. This is true only when x and y are both positive or both are negative. In the xy-plane, the only place where x and y are both positive and both negative is in the first and third quadrant. The domain of f(x,y) = ln(xy) is all pairs of points in the first and third quadrant of the xy-plane.

Suppose f(x,y) = $x^2 + y^2 - 9$. Then, in the xy-plane, $x^2 + y^2 = 9$ is a circle of radius 3.

We could have one of the following five conditions:

$$x^2 + y^2 = 9$$
$$x^2 + y^2 < 9$$
$$x^2 + y^2 \le 9$$
$$x^2 + y^2 > 9$$
$$x^2 + y^2 \ge 9$$

For every pair of points(x,y) on the circumference $x^2 + y^2 = 9$

For every pair of points(x,y) inside the circle $x^2 + y^2 < 9$

For every pair of points (x,y) outside the circle $x^2 + y^2 > 9$

Example: $x^2 + y^2 \le 9$. The domain is all pairs of points (x,y) inside the circle and on the circumference.

Example: f(x,y) = $\sqrt{9-\left(x^2+y^2\right)}$ $\quad x^2 + y^2 \le 9$

f(x,y) = $\sqrt{x^2+y^2-9}$ $\quad x^2 + y^2 \ge 9$

f(x,y) = $\dfrac{1}{\ln(xy)}$ $\quad xy > 0$

Example: z = f(x,y) = $\sqrt{9-\left(x^2+y^2\right)}$ $\quad x^2 + y^2 \le 9$

Since x^2 and y_2 will always be positive, we have $0 \le x^2 + y^2 \le 9$

so that $0 \le z \le 3$, since z= $\sqrt{9-\left(x^2+y^2\right)}$

The domain is all pairs of points(x,y) inside the circle or radius 3 and on the circumference. The range of f(x,y) is from 0 to 3.

Exercises: Find the domain of the following functions:

(1). $f(x,y) = x + 2y - 5$

(2). $f(x,y,z) = \ln(z - y) + xy \sin z$

(3). $f(x,y)=2/(x + y)$

(4). $f(x,y)= \varepsilon^{\left(x^2 - y\right)}$

(5). $f(x,y)= x^2 \ln(x - y + z)$

Answers:

(1). The whole xy-plane

(2). All x,y, and z provided $z > y$

(3). $x + y$ cannot equal zero

(4). The whole xy-plane

(5). $x - y + z$ must be greater than zero