Derivatives Involving Absolute Value

Calculating derivatives of functions containing absolute values can be confusing at first, but is actually relatively easy once we know the formula.

Suppose we want to find the derivative of $|\mathrm{u}|$. Recall that $|\mathrm{u}| = \sqrt{\mathrm{u}^2}$. So, we can calculate the derivative as follows:

$$\frac{d}{dx}|\mathrm{u}| = \frac{d}{dx}\sqrt{\mathrm{u}^2} = \frac{d}{dx}\left(\mathrm{u}^2\right)^{\frac{1}{2}} = \frac{1}{2}\left(\mathrm{u}^2\right)^{-\frac{1}{2}} \bullet 2\mathrm{u}\bullet\mathrm{u}' = \frac{\mathrm{u}}{\left(\mathrm{u}^2\right)^{\frac{1}{2}}}\bullet\mathrm{u}' = \frac{\mathrm{u}\bullet\mathrm{u}'}{\sqrt{\mathrm{u}^2}} = \frac{\mathrm{u}\bullet\mathrm{u}'}{|\mathrm{u}|}$$

Therefore, we have the following formula: $\boxed{\frac{d}{dx}|\mathrm{u}| = \frac{\mathrm{u}\bullet\mathrm{u}'}{|\mathrm{u}|},\ \mathrm{u} \neq 0}$

<u>Example 1:</u>
Find the first derivative of $f(x) = |x-1|$.

Solution:

Let $\mathrm{u} = x-1$. Then, following the formula, we get

$$\frac{d}{dx}|x-1| = \frac{(x-1)\bullet(x-1)'}{|x-1|} = \frac{x-1}{|x-1|}.$$

So, $f'(x) = \frac{x-1}{|x-1|}$.

Note that if x > 1, x – 1 is positive and $f'(x) = \frac{x-1}{x-1} = 1$. If x < 1, x – 1 is negative and $f'(x) = \frac{x-1}{-(x-1)} = -1$. $f'(x)$ does not exist at x = 1.

<u>Example 2:</u>
Find the first derivative of $f(x) = -x+2+|-x+2|$.

Solution:

Initially, for the derivative we get $f'(x) = -1 + \frac{d}{dx}|-x+2|$. Let $\mathrm{u} = -x+2$. Then, using the formula, we have $f'(x) = -1 + \frac{(-x+2)\bullet(-x+2)'}{|-x+2|} = -1 - \frac{-x+2}{|-x+2|}$.

So, $f'(x) = -1 - \frac{-x+2}{|-x+2|}$.

Note that if x > 2, $f'(x) = 0$, and if x < 2, $f'(x) = -2$. (Why?)
$f'(x)$ does not exist at x = 2.

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Exercises:

Find the first derivatives of the following functions.

1. $f(x) = |2x - 5|$
2. $g(x) = (x-2)^2 + |x-2|$

Answers:

1. $f'(x) = \dfrac{4x - 10}{|2x - 5|}$
2. $g'(x) = 2x - 4 + \dfrac{x-2}{|x-2|}$

Reference:
http://www.analyzemath.com/calculus/Differentiation/absolute_value.html