

MAT 218 – PRACTICE EXAM #5

Show all work neatly for partial credit. Circle your final answer.

1. (10 pts) Find $\int \left(4x^3 + \frac{2}{x^3} - 3 \right) dx$

2. (10 pts) Find $\int \frac{x^2 - 4x}{\sqrt{x}} dx$

3. (10 pts) Find $\int 3x^2 e^{x^3-4} dx$

4. (10 pts) Find $\int \frac{5(\ln x)^4}{2x} dx$

5. (10 pts) Find $\int_1^e \frac{1}{x} dx$

6. (10 pts) Find $\int_0^1 x^2(2x^3 - 1)^4 dx$

7. (10 pts) Find the area of the region under the graph of $f(x) = \sqrt[3]{x}$ on the interval $[1, 8]$.

8. (10 pts) The daily marginal revenue function associated with selling x units of a certain product is $R'(x) = -.1x + 40$ where $R'(x)$ is measured in dollars per unit. Find the daily total revenue from the sale of 200 units.
9. (10 pts) Roughly sketch the graphs of $y = x^2$ and $y = x$. Shade the region that is completely enclosed by the two graphs. Find the area of this region.

ANSWERS

$$\begin{aligned}
 1. \quad & \int \left(4x^3 + \frac{2}{x^3} - 3 \right) dx \\
 &= \int (4x^3 + 2x^{-3} - 3) dx \\
 &= x^4 - \frac{1}{x^2} - 3x + C
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \int \frac{x^2 - 4x}{\sqrt{x}} dx \\
 &= \int x^{\frac{3}{2}} - 4x^{\frac{1}{2}} dx \\
 &= \frac{2x^{\frac{5}{2}}}{5} - \frac{8x^{\frac{3}{2}}}{3} + C
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & \int 3x^2 e^{x^3-4} dx \\
 & \text{let } u = x^3 - 4, \quad du = 3x^2 dx \\
 & \int e^u du = e^u + C \\
 &= e^{x^3-4} + C
 \end{aligned}$$

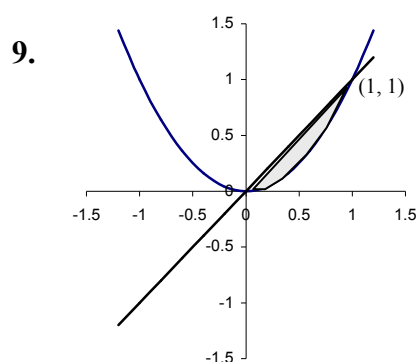
$$\begin{aligned}
 4. \quad & \int \frac{5(\ln x)^4}{2x} dx \\
 & \text{let } u = \ln x, \quad du = \frac{1}{x} dx \\
 & \int \frac{5u^4}{2} du = \frac{u^5}{2} + C \\
 &= \frac{(\ln x)^5}{2} + C
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & \int_1^e \frac{1}{x} dx \\
 &= \ln x \Big|_1^e \\
 &= \ln(e) - \ln(1) \\
 &= 1 - 0 \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & \int_0^1 x^2(2x^3 - 1)^4 dx \\
 & \text{let } u = 2x^3 - 1, \quad du = 6x^2 dx \\
 & \frac{1}{6} \int u^4 du \\
 &= \frac{u^5}{30} = \frac{(2x^3 - 1)^5}{30} \Big|_0^1 \\
 &= \frac{1}{30} + \frac{1}{30} = \frac{1}{15}
 \end{aligned}$$

$$7. \quad \int_1^8 \sqrt[3]{x} dx = \int_1^8 x^{\frac{1}{3}} dx = \frac{3}{4} x^{\frac{4}{3}} \Big|_1^8 = \frac{3}{4} (2^4 - 1) = \frac{45}{4} = 11.25$$

$$\begin{aligned}
 8. \quad R'(x) &= -.1x + 40 & \int_0^{200} -.1x + 40 dx &= -.05x^2 + 40x \Big|_0^{200} \\
 & & &= -.05(200)^2 + 40(200) = \$6,000
 \end{aligned}$$



$$\begin{aligned}
 \text{Area} &= \int_0^1 x - x^2 dx = \frac{x^2}{2} - \frac{x^3}{3} \Big|_0^1 \\
 &= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}
 \end{aligned}$$