

MAT 218 PRACTICE COMPREHENSIVE FINAL

Do not show your work. Place your final answer on the line provided. No partial credit will be given for this exam.

1. Find $\lim_{x \rightarrow \infty} \left(100 + \frac{50}{x} \right)$ 1. _____

2. Find $\lim_{x \rightarrow 0} \left(\frac{5x+1}{x} \right)$ 2. _____

3. List all x values for which the function $f(x) = \frac{x^2 - 2x - 3}{x^2 + x - 2}$ is discontinuous. 3. _____

4. Find the derivative of $f(x) = \frac{1+x}{(2x^2+1)^2}$ 4. _____

5. Find the derivative of $f(x) = 2x^3(x^2+1)^4$ 5. _____

6. What is the slope of the tangent line to the graph of $f(x) = 2x^3 - 3x^2 - 16x + 3$ at the point $(0, 3)$? 6. _____

7. Differentiate $3x^2 - y = 4$ with respect to t and find the value of $\frac{dy}{dt}$ when $x = 1$ and $\frac{dx}{dt} = 2$. 7. _____

8. If the cost function is $C(x) = 2x^3 - 4\sqrt{x}$ and the revenue function is $R(x) = x^4 + 5$, find the marginal profit when $x = 1$. 8. _____

9. Find all relative extrema (if any) of $f(x) = x^3 - 3x + 6$. Give your answers as points and identify them as maxima or minima. 9. _____

10. Determine the intervals for which the function $f(x) = x^3 - 2$ is concave up and concave down. 10. _____

For problems 11, 12, and 13, refer to the function f , given below along with its first and second derivatives.

$$f(x) = \frac{x+1}{(x-1)^2}$$

$$f'(x) = \frac{-x-3}{(x-1)^3}$$

$$f''(x) = \frac{2(x+5)}{(x-1)^4}$$

11. Write the equations of any vertical and/or horizontal asymptotes of the graph of $f(x)$. 11. _____

Give the x and y intercepts (if any). _____

12. Determine the intervals for which $f(x)$ is increasing and decreasing. 12. _____

Does $f(x)$ have any relative extrema? _____

If so, list the x -values and identify them as maxima or minima. _____

13. Determine the intervals for which $f(x)$ is concave up and concave down. 13. _____

Does $f(x)$ have any inflection points? _____

If so, list the x -values where they occur. _____

14. Find the absolute minimum value of $f(x) = x^3 + 3x^2 - 1$ on the interval $[-3, 1]$. 14. _____

15. Find the derivative of $f(x) = e^{4x} \ln(x^2)$ 15. _____

16. A hotel was purchased for \$4.1 million and sold 6 years later for \$8.2 million. Find the annual rate of return (compounded continuously) 16. _____

17. Find $\int \frac{x}{3x^2 + 1} dx$ 17. _____

18. Find $\int 3te^{2t^2} dt$ 18. _____

19. Find $\int_{-1}^1 x^2(x^3 + 1)^4 dx$ 19. _____

20. Find the area bounded by $y = x^3$, $x = -2$,
 $x = 1$, and $y = 0$. 20. _____

ANSWERS

1. 100
2. Does not exist
3. -2, 1
4. $\frac{-6x^2 - 8x + 1}{(2x^2 + 1)^3}$
5. $2x^2(x^2 + 1)^3(11x^2 + 3)$
6. -16
7. 12
8. 0
9. max (-1, 8) min (1, 4)
10. concave up $(0, \infty)$
concave down $(-\infty, 0)$
11. V.A. $x = 1$; H.A. $y = 0$
x-intercept $(-1, 0)$; y-intercept $(0, 1)$
12. increasing $(-3, 1)$
decreasing $(-\infty, -3)$ and $(1, \infty)$
yes: relative min at $x = -3$
13. concave down $(-\infty, -5)$
concave up $(-5, 1)$ and $(1, \infty)$
yes; $x = -5$
14. -1
15. $\frac{2e^{4x}}{x} + 4e^{4x} \ln(x^2)$
or $\frac{2e^{4x}}{x} + 8e^{4x} \ln x$
16. 11.55%
17. $\frac{\ln(3x^2 + 1)}{6} + C$
18. $\frac{3e^{2t^2}}{4} + C$
19. $\frac{32}{15}$
20. $\frac{17}{4}$