

Solving for *Interest* in a Present Value or Future Value Problem With or Without a Graphing Calculator

Consider the equation:
$$PV = PMT \left[\frac{1 - (1 + i)^{-n}}{i} \right]$$

Given: PV = \$10,000 PMT = \$500 n = 30 **Solve for i.**

Step 1. Put known values into the equation.
$$10,000 = 500 \left[\frac{1 - (1 + i)^{-30}}{i} \right]$$

Step 2. Manipulate the equation to get all the i terms on one side equal to something.

$$\frac{10,000}{500} = \left[\frac{1 - (1 + i)^{-30}}{i} \right]$$

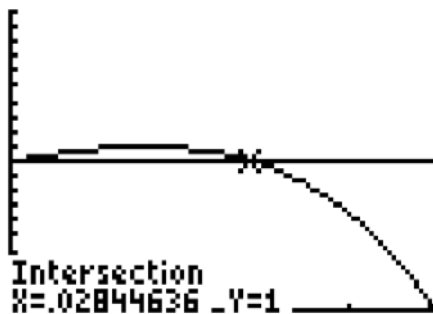
$$20 = \frac{1 - (1 + i)^{-30}}{i}$$

$$20i = 1 - (1 + i)^{-30}$$

$$1 - 20i = \frac{1}{(1 + i)^{30}}$$

$$(1 + i)^{30}(1 - 20i) = 1$$

Step 3. If you have a graphing calculator, plot $y = (1 + i)^{30}(1 - 20i)$ and $y = 1$ and find the intersection. Or, set your table to very small increments and find where $y = 1$ for your function, assuming $i > 0$.



X	Y ₁	Y ₂
.028	1.0075	1
.0281	1.0059	1
.0282	1.0042	1
.0283	1.0025	1
.0284	1.0008	1
.0285	.99907	1
.0286	.99733	1

X = .0284

Or Step 3. If you don't have a graphing calculator, you can still solve for i by iteration.

Notice that $(1 + i)^{30}$ is always positive because it has an even exponent (also, in this case, i is positive). Therefore $(1 - 20i)$ must also be positive for the product of the two to equal a positive number.

a. State that as:

$$\begin{aligned} 1 - 20i &> 0 \\ 20i &< 1 \\ i &< \frac{1}{20} \quad \text{or} \quad i < .05 \end{aligned}$$

b. With a scientific calculator, evaluate the function $y = (1 + i)^{30}(1 - 20i)$ for values of $i < .05$. The goal is to get the equation equal to 1.

i	$(1 + i)^{30}(1 - 20i)$
.04	0.65
.03	0.97
.02	1.087
.029	0.9902
.028	1.0075
.0284	1.0008
etc.	depending on how close you need to get

$$i \approx 2.84\%$$

General form to solve either $PV = PMT \left[\frac{1 - (1 + i)^{-n}}{i} \right]$ or $PMT = PV \left[\frac{i}{1 - (1 + i)^{-n}} \right]$ for i

$$\text{is} \quad (1 + i)^n \left[1 - \left(\frac{PV}{PMT} \right) i \right] = 1 \quad i < \frac{PMT}{PV}$$

General form to solve either $FV = PMT \left[\frac{(1 + i)^n - 1}{i} \right]$ or $PMT = FV \left[\frac{i}{(1 + i)^n - 1} \right]$ for i

$$\text{is} \quad (1 + i)^n - \left(\frac{FV}{PMT} \right) i = 1$$

(Note: In solving the FV equation for i with a scientific calculator, there is no defined boundary for i , so just start the iterations with $i = .01$. You're still trying to get the left side of the equation equal to 1.)