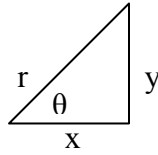


Trigonometric Identities

Basic Identities

$$\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$



$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{r}{y} \quad \sec \theta = \frac{r}{x}$$

$$\cot \theta = \frac{x}{y}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}$$

$$\sin \theta = \cos (90^\circ - \theta)$$

$$\cos \theta = \sin (90^\circ - \theta)$$

$$\tan \theta = \cot (90^\circ - \theta)$$

$$\sin (-\theta) = -\sin \theta$$

$$\cos (-\theta) = \cos \theta$$

$$\tan (-\theta) = -\tan \theta$$

$$\sin (\theta + \pi) = -\sin \theta$$

$$\cos (\theta + \pi) = -\cos \theta$$

$$\tan (\theta + \pi) = \tan \theta$$

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sec^2 \theta - \tan^2 \theta = 1$$

$$\csc^2 \theta - \cot^2 \theta = 1$$

Double- and Triple-Angle Identities

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$$

$$\cos 2\theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$$

$$\tan 3\theta = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$$

Half-Angle Identities

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\tan \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \frac{1 - \cos \theta}{\sin \theta} = \frac{\sin \theta}{1 + \cos \theta} = \csc \theta - \cot \theta$$

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

Sums and Differences of Angles

$$\sin (A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin (A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos (A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos (A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\cot (A + B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$$

$$\cot (A - B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

Product and Sum Identities

$$\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$$

$$\cos A \cos B = \frac{1}{2} [\cos(A - B) + \cos(A + B)]$$

$$\sin A \cos B = \frac{1}{2} [\sin(A + B) + \sin(A - B)]$$

$$\cos A \sin B = \frac{1}{2} [\sin(A + B) - \sin(A - B)]$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Powers of Trig Functions

$$\sin^2 \theta = \frac{1}{2} - \frac{1}{2} (\cos 2\theta)$$

$$\cos^2 \theta = \frac{1}{2} + \frac{1}{2} (\cos 2\theta)$$

$$\tan^2 \theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

$$\sin^3 \theta = \frac{3}{4} (\sin \theta) - \frac{1}{4} (\sin 3\theta)$$

$$\cos^3 \theta = \frac{3}{4} (\cos \theta) + \frac{1}{4} (\cos 3\theta)$$

Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

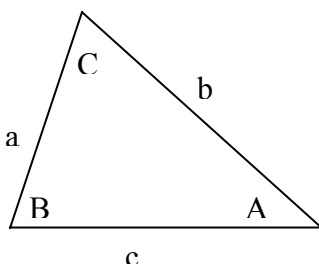
or $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Angles Unknown

$$\sin A = \frac{2}{bc} \sqrt{s(s-a)(s-b)(s-c)}$$

Law of Cosines

$$c^2 = a^2 + b^2 - 2ab \cos C$$



Law of Tangents

$$\frac{a+b}{a-b} = \frac{\tan \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)}$$

Area

$$\begin{aligned} A &= \frac{1}{2} (bc) \sin A \\ &= \frac{1}{2} (ab) \sin C \\ &= \frac{1}{2} (ac) \sin B \end{aligned}$$

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{1}{2} (a + b + c)$$

Arc length of sector of a circle: $s = r\theta$

Area of sector: $A = \frac{1}{2} r^2 \theta$

[θ in radians]

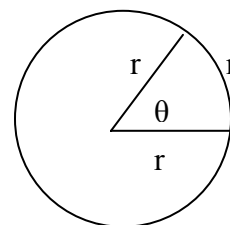
Degrees and Radians

One radian = angle θ subtended by an arc equal to the radius
= approx. 57.3°

$$2\pi \text{ radians} = 360^\circ$$

$$1 \text{ radian} = \frac{360^\circ}{2\pi} = \frac{180^\circ}{\pi}$$

$$1^\circ = \frac{\pi}{180} \text{ radians}$$



□