

## Products, Quotients, Powers, and Roots of Complex Numbers (Worksheet for VOM Tape A-35)

1. Forms of complex numbers.

Rectangular		Polar		Exponential
$a + bj$	=	$r(\cos \theta + j \sin \theta)$	=	$re^{j\theta}$
		where $r = \sqrt{a^2 + b^2}$		$\theta$ is in radians
		and $\theta$ is computed		
		from $\tan \theta = \frac{b}{a}$		

2. Products of complex numbers in polar form.

$$a(\cos \alpha + j \sin \alpha) \cdot b(\cos \beta + j \sin \beta) = ab[\cos(\alpha + \beta) + j \sin(\alpha + \beta)]$$

3. Quotients of complex numbers in polar form.

$$\frac{a(\cos \alpha + j \sin \alpha)}{b(\cos \beta + j \sin \beta)} = \frac{a}{b} [\cos(\alpha - \beta) + j \sin(\alpha - \beta)]$$

4. DeMoivre's Theorem.

$$[r(\cos \theta + j \sin \theta)]^n = r^n (\cos n\theta + j \sin n\theta), \quad n \text{ is any real number}$$

### Examples

**A. Find the product of  $(-3 + 3j\sqrt{3})(2\sqrt{3} + 2j)$  in polar form.**

Method

- |  |   |
|--|---|
| 1. Change to polar form.   | $(-3 + 3j\sqrt{3})(2\sqrt{3} + 2j)$ $= 6(\cos 120^\circ + j \sin 120^\circ) \cdot 4(\cos 30^\circ + j \sin 30^\circ)$ |
| 2. Multiply coefficients,<br>add angles and write in polar form. | $= 6 \cdot 4[\cos(120^\circ + 30^\circ) + j \sin(120^\circ + 30^\circ)]$ $= 24(\cos 150^\circ + j \sin 150^\circ)$    |
| 3. Write in rectangular form (if desired).                       | $= 24 \left( -\frac{\sqrt{3}}{2} + \frac{1}{2}j \right) = -12\sqrt{3} + 12j$  |

NOTE: Result could be checked by multiplying in rectangular form.

**B. Find the quotient of  $\frac{-4\sqrt{3}+4j}{\sqrt{3}+j}$  in polar form.**

Method

1. Change to polar form.  $\frac{-4\sqrt{3}+4j}{\sqrt{3}+j} = \frac{8(\cos 150^\circ + j\sin 150^\circ)}{2(\cos 30^\circ + j\sin 30^\circ)}$
2. Divide coefficients and subtract the angle of the denominator from the angle of the numerator.  $= 4(\cos 120^\circ + j\sin 120^\circ)$
3. Write in rectangular form (if desired).  $= -2 + 2j\sqrt{3}$   
NOTE: Result could be checked by dividing in rectangular form.

**C. Raise to the sixth power  $-2.4575 + 1.7207j$**

Method

1. Change to polar form.  $(-2.4575 + 1.7207j)^6$   
 $= [3(\cos 145^\circ + j\sin 145^\circ)]^6$
2. Raise the coefficient to the power and multiply the angle by the exponent.  $= 3^6(\cos 6 \cdot 145^\circ + j\sin 6 \cdot 145^\circ)$   
 $= 729(\cos 870^\circ + j\sin 870^\circ)$
3. Convert to coterminal angle less than  $360^\circ$ .  $= 729(\cos 150^\circ + j\sin 150^\circ)$
4. Write in rectangular form (if desired).  $= 729\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}j\right)$   
 $= -631.33 + 364.5j$

**D Find all roots of  $z^4 = -312.5 - 541.265 j$**

Method

- Write in polar form.  $z^4 = 625(\cos 240^\circ + j \sin 240^\circ)$
- Set up a table, where  $\text{cjs } \theta = \cos \theta + j \sin \theta$

3. Find 4 <sup>th</sup> root of the coefficient and divide the angle by the exponent to find the angle of the first root.	$625 \text{ cjs } 240^\circ$	$Z_1 = \sqrt[4]{625} \text{ cjs } \frac{240^\circ}{4}$ $= 5 \text{ cjs } 60^\circ$
4. To get subsequent roots, add multiples of $360^\circ$ to the angle, then divide by the exponent.	$625 \text{ cjs } (240^\circ + 360^\circ)$ $= 625 \text{ cjs } 600^\circ$	$Z_2 = 5 \text{ cjs } 150^\circ$
	$625 \text{ cjs } (240^\circ + 2 \cdot 360^\circ)$ $= 625 \text{ cjs } 960^\circ$	$Z_3 = 5 \text{ cjs } 240^\circ$
	$625 \text{ cjs } (240^\circ + 3 \cdot 360^\circ)$ $= 625 \text{ cjs } 1320^\circ$	$Z_4 = 5 \text{ cjs } 330^\circ$

- Write roots in polar form and change to rectangular form, if desired.

**Problems**

Find the products of the following in polar form.

- $3(\cos 45^\circ + j \sin 45^\circ) \cdot 4(\cos 100^\circ + j \sin 100^\circ)$
- $(1 + j)(-1 - j)$

Find the powers of the following.

- $(-3 + 3j)^3$
- $(3 + 4j)^4$

Find the quotients of the following.

- $\frac{3\sqrt{2}(\cos 45^\circ + j \sin 45^\circ)}{\sqrt{2}(\cos 30^\circ + j \sin 30^\circ)}$
- $\frac{3 - 3j}{-2 + 2j}$

Find the roots of the following.

- $Z^3 = -4 + 4j\sqrt{3}$
- $Z^4 = -81$

## Answers

1.  $12(\cos 145^\circ + j \sin 145^\circ)$

2.  $-2j$

3.  $2.90 + .78j$

4.  $-\frac{3}{2}$

5.  $54 + 54j$

6.  $-527 - 336j$

7.  $Z_1 = 2(\cos 40^\circ + j \sin 40^\circ) = 1.532 + 1.286j$

$$Z_2 = 2(\cos 160^\circ + j \sin 160^\circ) = -1.879 + 0.684j$$

$$Z_3 = 2(\cos 280^\circ + j \sin 280^\circ) = 0.347 - 1.970j$$

8.  $Z_1 = 3(\cos 45^\circ + j \sin 45^\circ) = 2.12 + 2.12j$

$$Z_2 = 3(\cos 135^\circ + j \sin 135^\circ) = -2.12 + 2.12j$$

$$Z_3 = 3(\cos 225^\circ + j \sin 225^\circ) = -2.12 - 2.12j$$

$$Z_4 = 3(\cos 315^\circ + j \sin 315^\circ) = 2.12 - 2.12j$$