

# POLAR AND EXPONENTIAL FORMS OF A COMPLEX NUMBER

## (Worksheet for VOM Tape A-34)

### Forms of Complex Numbers

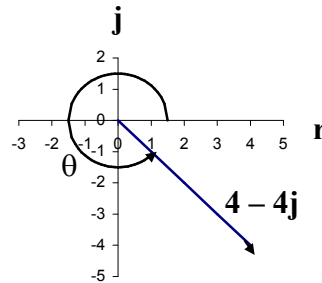
Rectangular	=	Polar	=	Exponential
$a + bj$		$r(\cos \theta + j \sin \theta)$		$re^{j\theta}$
		where $r = \sqrt{a^2 + b^2}$ and $\theta$ is computed from $\tan \theta = \frac{b}{a}$		$\theta$ is in radians

### Examples:

**A. Change  $4 - 4j$  to polar and exponential forms.**

#### Method

1. Draw graphical representation of the complex number.



2. Compute  $r$ .

$$\begin{aligned}
 r &= \sqrt{4^2 + (-4)^2} \\
 &= \sqrt{16 + 16} = \sqrt{32} \\
 &= 4\sqrt{2}
 \end{aligned}$$

3. Compute  $\theta$ .

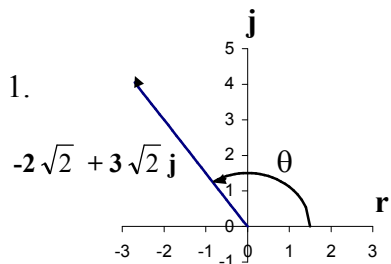
$$\tan \theta = \frac{-4}{4} = -1$$

$$\theta = 315^\circ = \frac{7\pi}{4} \text{ rad (fourth quadrant)}$$

#### Solution

Rectangular form	=	Polar form	=	Exponential form
$4 - 4j$		$4\sqrt{2} (\cos 315^\circ + j \sin 315^\circ)$		$4\sqrt{2} e^{j\frac{7\pi}{4}}$
		or $4\sqrt{2} (\cos \frac{7\pi}{4} \text{ rad} + j \sin \frac{7\pi}{4} \text{ rad})$		

**B. Change  $-2\sqrt{2} + 3\sqrt{2}j$  to polar and exponential forms.**



2. 
$$r = \sqrt{(-2\sqrt{2})^2 + (3\sqrt{2})^2}$$

$$= \sqrt{8+18} = \sqrt{26}$$

3. 
$$\tan \theta = \frac{3\sqrt{2}}{-2\sqrt{2}} = -1.5$$

$$\theta = 123.7^\circ = 2.16 \text{ rad}$$

Solution

**Rectangular form**

**Polar form**

**Exponential form**

$$-2\sqrt{2} + 3\sqrt{2}j = \sqrt{26} (\cos 123.7^\circ + j \sin 123.7^\circ) = \sqrt{26} e^{2.16j}$$

or  $\sqrt{26} (\cos 2.16 \text{ rad} + j \sin 2.16 \text{ rad})$

**C. Change  $8e^{3.9j}$  to rectangular form.**

Method

- |                                                     |                                        |
|-----------------------------------------------------|----------------------------------------|
| 1. Change to polar form ( $\theta$ in radians).     | $8e^{3.9j} = 8(\cos 3.9 + j \sin 3.9)$ |
| 2. Find sine and cosine values for the given angle. | $= 8(-0.7259 - 0.6878j)$               |
| 3. Multiply through.                                | $= -3.63 - 5.50j$                      |

Solution

**Exponential form**

**Polar form**

**Rectangular form**

$$8e^{3.9j} = 8(\cos 3.9 + j \sin 3.9) = -3.63 - 5.50j$$

**Additional Problems**

- Change to polar and exponential forms: 1)  $-3 - 2j$       2)  $2\sqrt{3} - 2j$
- Change to rectangular form: 3)  $15(\cos 120^\circ + j \sin 120^\circ)$     4)  $12(\cos \frac{\pi}{3} + j \sin \frac{\pi}{3})$     5)  $6 e^{\frac{4\pi j}{3}}$

**Answers**

1.  $\sqrt{13} (\cos 213.7^\circ + j \sin 213.7^\circ)$  or  $\sqrt{13} (\cos 3.73 \text{ rad} + j \sin 3.73 \text{ rad})$ ;  $\sqrt{13} e^{3.73j}$
2.  $4(\cos 330^\circ + j \sin 330^\circ)$  or  $4(\cos \frac{5\pi}{3} \text{ rad} + j \sin \frac{5\pi}{3} \text{ rad})$ ;  $4 e^{\frac{5\pi j}{3}}$
3.  $-7.5 + 13.0j$       4.  $6 + 10.4j$       5.  $-3 - 5.02j$