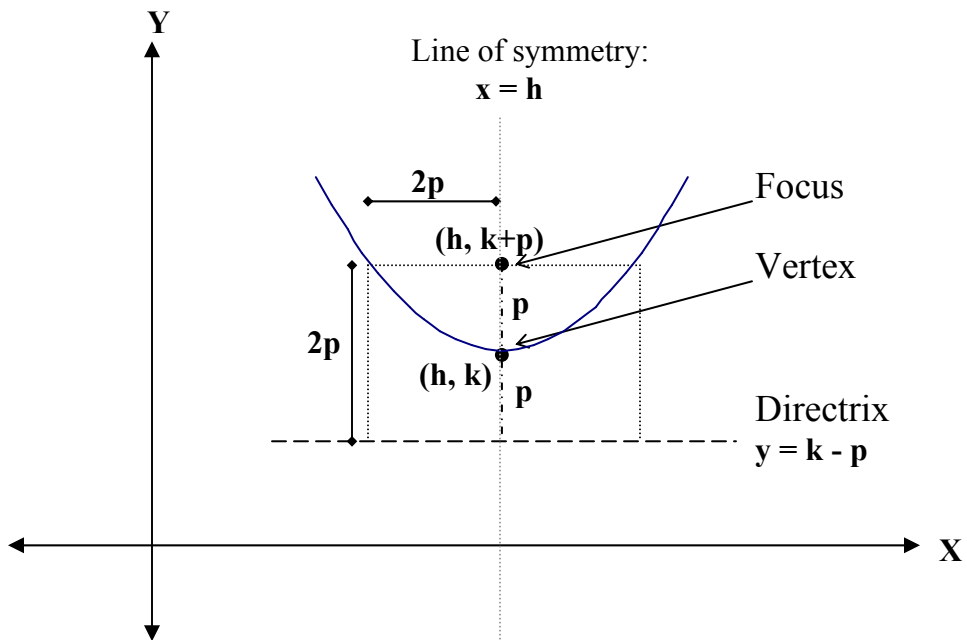


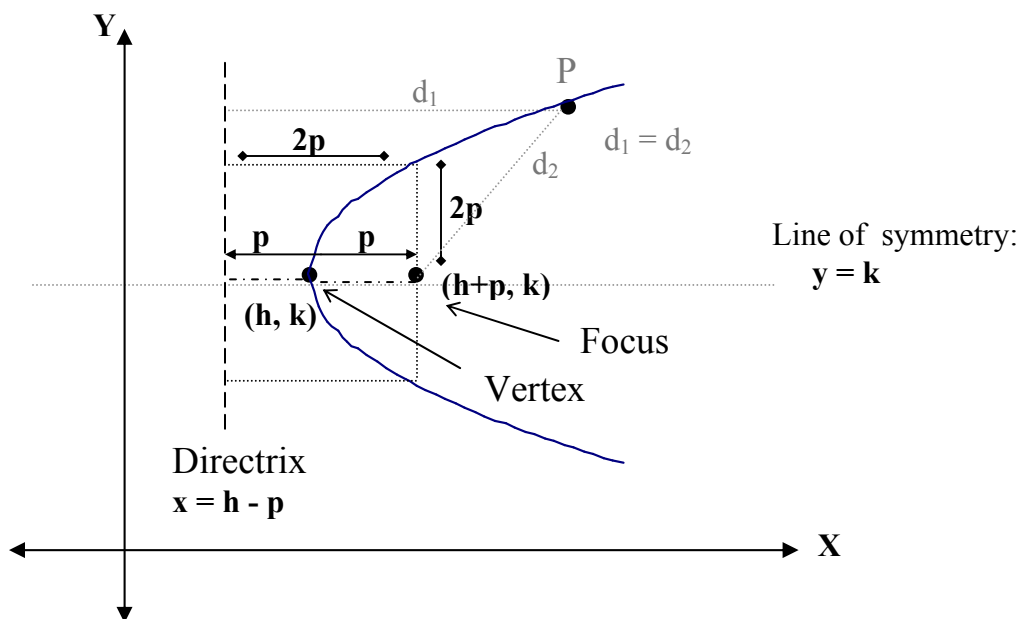
**CONIC SECTIONS:**

**PARABOLA**



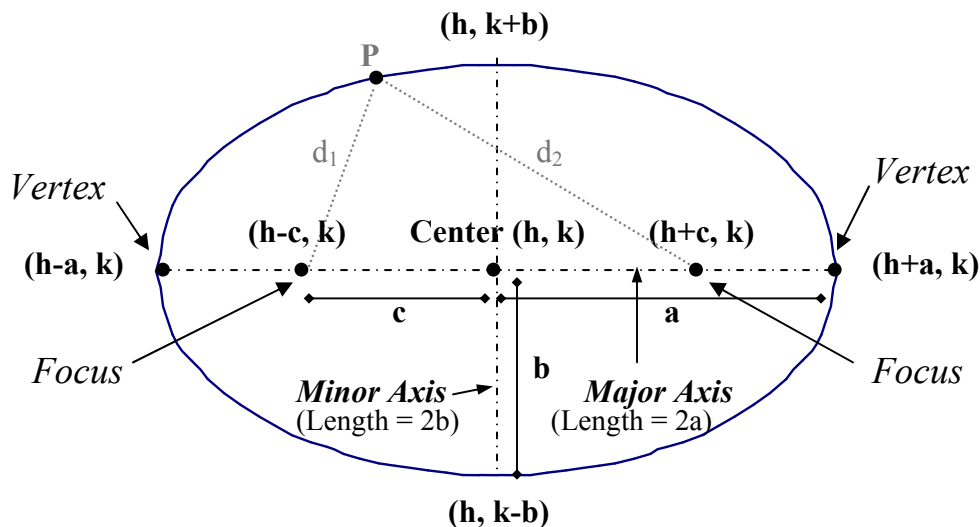
	Standard Equation	Vertex	Focus	Directrix
Vertical	$(x - h)^2 = 4p(y - k)$	$(h, k)$	$(h, k + p)$	$y = k - p$
Horizontal	$(y - k)^2 = 4p(x - h)$	$(h, k)$	$(h + p, k)$	$x = h - p$

**Every point on a parabola is the same distance from the focus as from the directrix.**



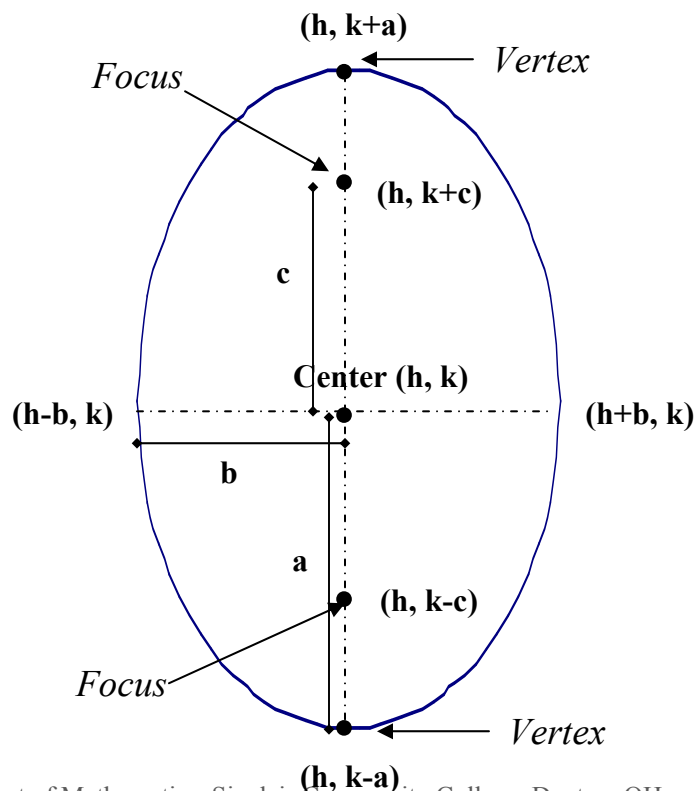
# CONIC SECTIONS:

# ELLIPSE



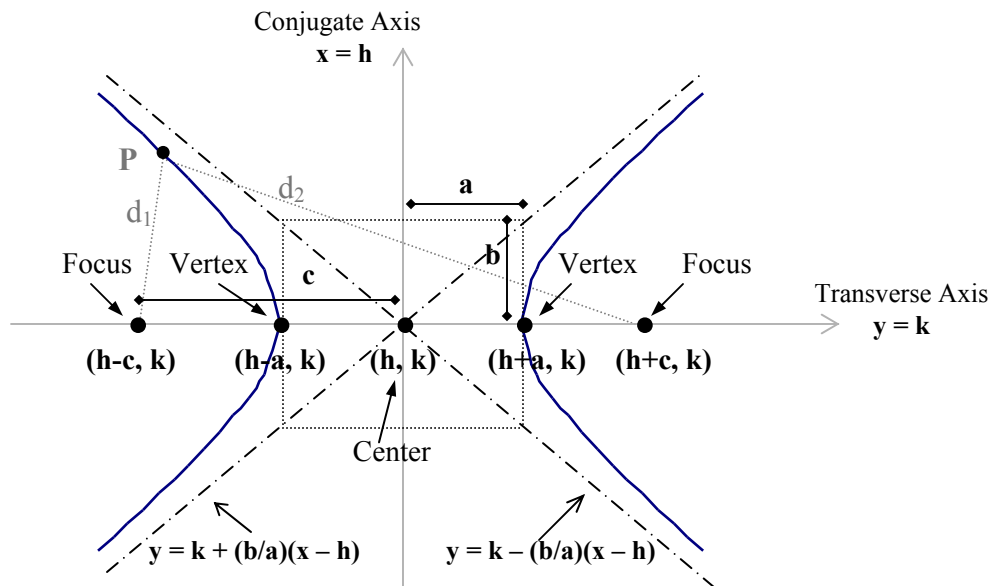
	Standard Equation	Center	Foci	Vertices
Horizontal	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$(h, k)$	$(h+c, k), (h-c, k)$	$(h+a, k), (h-a, k)$
Vertical	$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$	$(h, k)$	$(h, k+c), (h, k-c)$	$(h, k+a), (h, k-a)$
Eccentricity: $e = \frac{c}{a} < 1$	$a > b$		$c^2 = a^2 - b^2$ (graphs not to scale)	

For any point P on an ellipse, the sum of the distances from P to the foci is a constant:  $d_1 + d_2 = 2a$ .  
If  $a = b$ , then  $c = 0$ , and the ellipse is a circle.



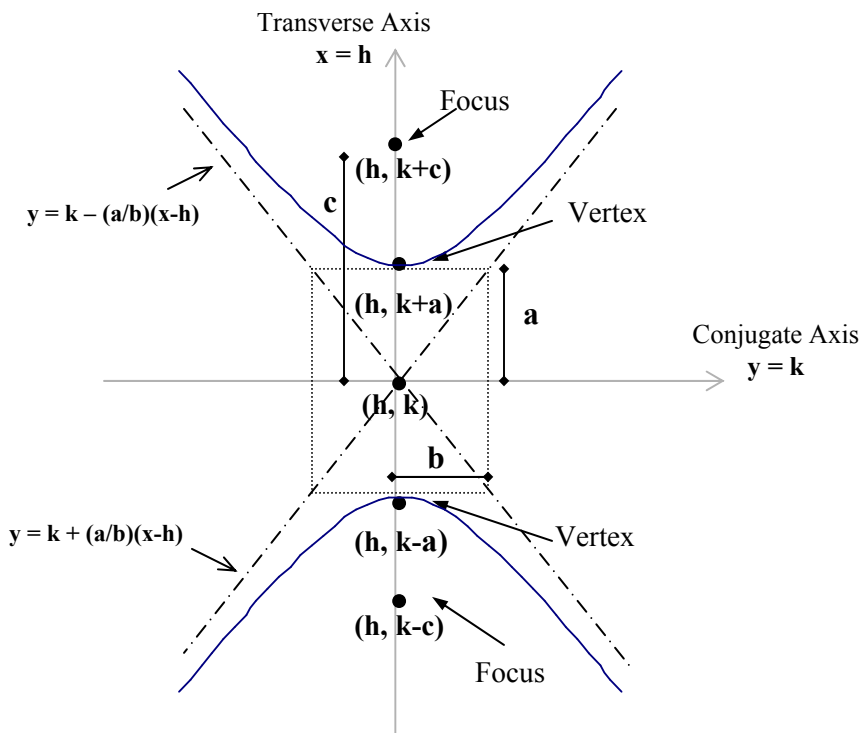
# CONIC SECTIONS:

# HYPERBOLA

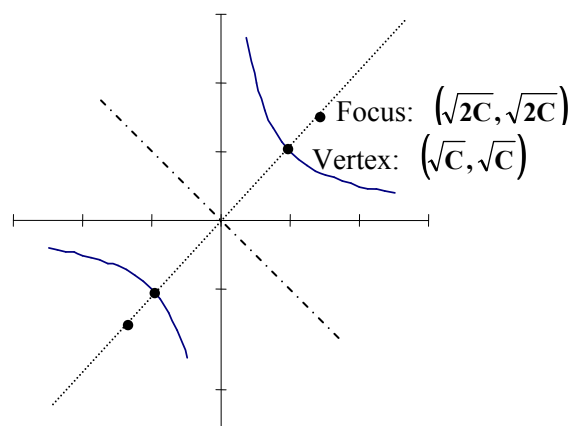


	Standard Equation	Center	Foci	Vertices	Slant Asymptotes
Transverse axis parallel to x-axis	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$(h, k)$	$(h+c, k), (h-c, k)$	$(h+a, k), (h-a, k)$	$y = k \pm \left(\frac{b}{a}\right)(x-h)$
Transverse axis parallel to y-axis	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$	$(h, k)$	$(h, k+c), (h, k-c)$	$(h, k+a), (h, k-a)$	$y = k \pm \left(\frac{a}{b}\right)(x-h)$
Eccentricity: $e = \frac{c}{a} > 1$			$c^2 = a^2 + b^2$ (graphs not to scale)		

For any point P on a hyperbola, the difference of the distances of P to the foci is a constant:  $d_2 - d_1 = 2a$



Another important type of hyperbola is defined by the equation  $xy = C$ , where  $C$  is a constant. The asymptotes are the  $x$  and  $y$  axes. The function is symmetric about the origin, and the transverse axis is  $y = x$ . (If  $C$  is negative, the transverse axis is  $y = -x$ , and the graph is in Quadrants II and IV.) This graph can also be viewed as a standard,  $x$ - $y$  oriented hyperbola that has been rotated  $45^\circ$  into a new coordinate system.



## THE SECOND-DEGREE EQUATION

The ellipse/circle, parabola, and hyperbola functions are all cases of the second-degree equation:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0.$$

	$Ax^2$	+	$Bxy$	+	$Cy^2$	+	$Dx$	+	$Ey$	+	$F$
	<b>A</b>		<b>B</b>		<b>C</b>		<b>D</b>		<b>E</b>		<b>F</b>
<b>Circle</b>	$= C$		$= 0$		$= A$						
<b>Ellipse</b>	$\neq C$		$= 0$		$\neq A$						
<b>Parabola</b>	$= 0$ if $C \neq 0$		$= 0$		$= 0$ if $A \neq 0$						
<b>Hyperbola</b>	<b>opp. sign of C</b>		$= 0$		<b>opp. sign of A</b>						
<b>Center/vertex not at origin</b>							$\neq 0$ and/or $\neq 0$				
<b>Rotated axis</b>			$\neq 0$								

To rotate axes (eliminate the  $xy$  term):

$\theta$  = angle of rotation from  $x$ -axis

$$\cot 2\theta = (A - C)/B$$

$$x = x' \cos\theta - y' \sin\theta$$

$$y = x' \sin\theta + y' \cos\theta$$

$$\text{Substitute } \rightarrow A'(x')^2 + C'(y')^2 + D'x' + E'y' + F' = 0$$

$$\text{Invariants: } F' = F$$

$$A + C = A' + C'$$

$$B^2 - 4AC = (B')^2 - 4A'C'$$

$$B^2 - 4AC < 0 \rightarrow \text{ellipse/circle}$$

$$B^2 - 4AC = 0 \rightarrow \text{parabola}$$

$$B^2 - 4AC > 0 \rightarrow \text{hyperbola}$$