10.2 Deduction

Definition: Deductive reasoning proceeds step by step from assumed statements called premises or hypotheses to a statement called a conclusion

Definition: A valid argument is an argument in which the conclusion must be true whenever the premises are true

Logical Arguments

Modus Ponens [Affirming the antecedent]

Premises: If p then q $\quad p \Rightarrow q$

p $\quad p$

Conclusion: Therefore q $\quad \therefore q$

Example: If you are at least 18 years old then you can register to vote
You are 30 years old
Therefore, you can register to vote

		Premises		Conclusion
p	q	$p \Rightarrow q$	p	q
T	T	T	T	T
T	F	F	T	F
F	T	T	F	T
F	F	T	F	F

The table above proves Modus Ponens is a valid argument form. It shows that whenever the premises are true (1^{st} row) the conclusion is true

Chain Rule

Premises: If p then q $\quad p \Rightarrow q$

If q then r $\quad q \Rightarrow r$

Conclusion: Therefore, If p then r $\quad \therefore p \Rightarrow r$

Example: If you are at least 65 years old then you can retire
If you are retired then you can golf every day
Therefore, If you are at least 65 years old then you can golf every day

			Premises		Conclusion
p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$p \Rightarrow r$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

The table above proves the Chain Rule is a valid argument form. It shows that whenever the premises are true (1^{st}, 5^{th}, 7^{th}, 8^{th} row) the conclusion is true

Modus Tollens [Law of contraposition, Denying the consequent]

Premises: If p then q $p \Rightarrow q$
Not q $\sim q$

Conclusion: Therefore not p $\therefore \sim p$

Example: If it raining then you will have your umbrella
You do not have your umbrella
Therefore, it is not raining

		Premises		Conclusion
p	q	$p \Rightarrow q$	$\sim q$	$\sim p$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

The table above proves Modus Tollens is a valid argument form. It shows that whenever the premises are true (4^{th} row) the conclusion is true

Note: It can be shown that Modus Ponens and Modus Tollens are logically equivalent

Disjunctive Syllogism [Law of the Excluded Middle]

Premises:	p or q	$p \vee q$
	Not q	$\sim q$
Conclusion:	Therefore p	$\therefore p$

Example: Today is Tuesday or Wednesday
Today is not Wednesday
Therefore, today is Tuesday

		Premises		Conclusion
p	q	$p \vee q$	$\sim q$	p
T	T	T	F	T
T	F	T	T	T
F	T	T	F	F
F	F	F	T	F

The table above proves Disjunctive Syllogism is a valid argument form. It shows that whenever the premises are true (2^{nd} row) the conclusion is true

Note: Disjunctive Syllogism is related to an often-used mathematical method of proof called argument by contradiction or reductio ad absurdum. With an argument by contradiction you assume the conclusion is false (you are going to try to prove it is true). Then you find a valid argument that leads to a contradiction. Since the conclusion must be true of false and since you know it is not false, it must be true.

Incorrect Reasoning Patterns

Fallacy of Affirming the Consequent [Misuse of Modus Ponens]

Premises:	If p then q	$p \Rightarrow q$
	q	q
Conclusion:	Therefore p	$\therefore p$

Example: If you smoke then you will get lung cancer
You got lung cancer
Therefore, you smoked

	Premises			Conclusion
p	q	$p \Rightarrow q$	q	p
T	T	T	T	T
T	F	F	F	T
F	T	T	T	***F***
F	F	T	F	F

The table above proves Affirming the Consequent is a **NOT** a valid argument form. In the 3rd row the premises are true but the conclusion is NOT true. (It does not matter that in the 1st row the premises were true and the conclusion was true)

Fallacy of Denying the Antecedent [Misuse of Modus Tollens]

Premises: If p then q $p \Rightarrow q$
Not p $\sim p$

Conclusion: Therefore not q $\therefore \sim q$

Example: If you smoke then you will get lung cancer
You do not smoke
Therefore, you will not get lung cancer

	Premises			Conclusion
p	q	$p \Rightarrow q$	$\sim p$	$\sim q$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	***F***
F	F	T	T	T

The table above proves Denying the Antecedent is a **NOT** a valid argument form. In the 3rd row the premises are true but the conclusion is NOT true. (It does not matter that in the 4th row the premises were true and the conclusion was true)

Note: The two fallacies are logically equivalent

The notes above are for Math 108, Math for the Modern World using

Mathematics in Life, Society and the World 2nd edition by Parks, Musser, Burton, and Siebler. Prentice Hall 2000.