

MAT 102
TEST #4
Review Sheet

All answers involving complex numbers should be written in the standard form $a + bi$.
Denominators are to be rationalized.

1. Solve by the square root property.

a) $x^2 = 24$

b) $5z^2 - 200 = 0$

c) $(m - 7)^2 = -\frac{3}{2}$

2. Solve by completing the square.

a) $x^2 - 3x + 7 = 0$

b) $4x^2 + 12x - 7 = 0$

c) $2x^2 - 11x + 1 = 0$

3. Solve using the quadratic formula.

a) $4x^2 + 4x = 1$

b) $m(m - 2) = -4$

4. Determine the nature of the solutions of the equations. Do not actually solve.

a) $x^2 - 2 = 0$

b) $x^2 - 2x + 4 = 0$

5. Find the value of a so that the equation will have exactly one rational solution.

$$at^2 + 24t + 16 = 0$$

6. Solve: $\frac{3}{3x+1} + \frac{6x}{11x-1} = 1$

7. Solve using the appropriate substitution.

a) $9\left(\frac{x+2}{x+3}\right)^2 - 6\left(\frac{x+2}{x+3}\right) + 1 = 0$

b) $3x^{2/3} - x^{1/3} - 24 = 0$

8. Two pipes together can fill a large tank in 2 hours. One of the pipes, used alone, takes 3 hours longer than the other to fill the tank. How long would each pipe take to fill the tank alone?

9. Working together, James and Kory can paint a certain length of standard size fence in 3 hours. It takes Kory 1 hour longer to paint the same fence than James. How long does it take James and Kory each separately to paint the fence.
10. Lekesha flew her plane for 6 hours at a constant speed. She traveled 810 miles with the wind, then turned around and traveled 720 miles against the wind. The wind speed was a constant 15 miles per hour. Find the speed of the plane.
11. The speed of a river is 2 mph. Jim travels in a canoe 4 miles downstream, stops to eat and rest for one hour, and then travels 4 miles back upstream. The total trip time is 5 hours. What is the speed of Jim's canoe in still water?
12. The hypotenuse of a right triangle is 25 ft. long. One leg is 17 ft. longer than the other. Find the lengths of the legs.
13. Solve the equation for t.

$$-B = 2t^2 - 6at + 3$$

14. Solve the equation for h .

$$L = \frac{kd^4}{h^2}$$

15. Solve the equation for g .

$$p = \sqrt{\frac{kl}{g}}$$

16. A rectangle has a length 2 meters less than twice its width. When 5 meters are added to the width, the resulting figure is a square with an area of 144 square meters. Find the dimensions of the original rectangle.

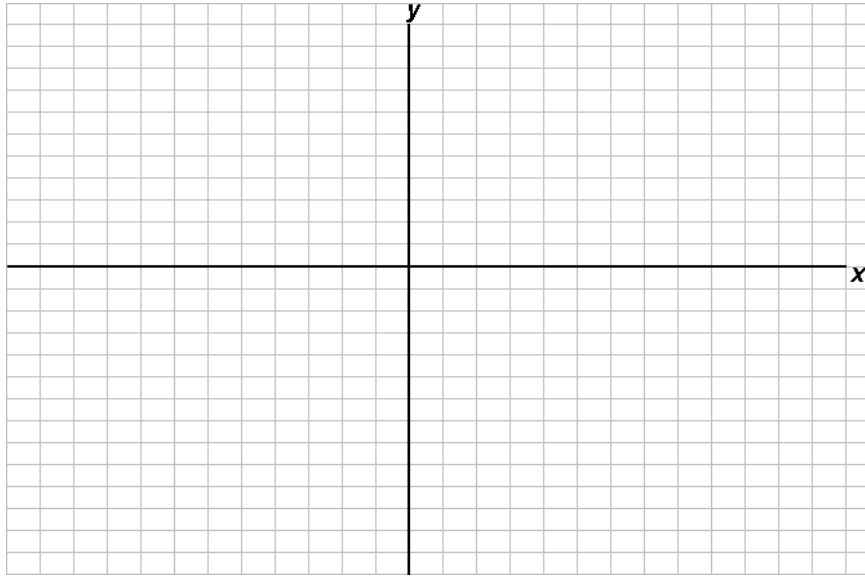
17. A toy rocket is launched from ground level. Its distance in feet from the ground in t sec is given by $s(t) = -16t^2 + 208t$. At which times will the rocket be 550 ft from the ground?

18. Without graphing, find the vertex, the axis of symmetry, the maximum or minimum value and the range. Decide whether the graph is wider, narrower or the same shape as the graph of $y = x^2$.

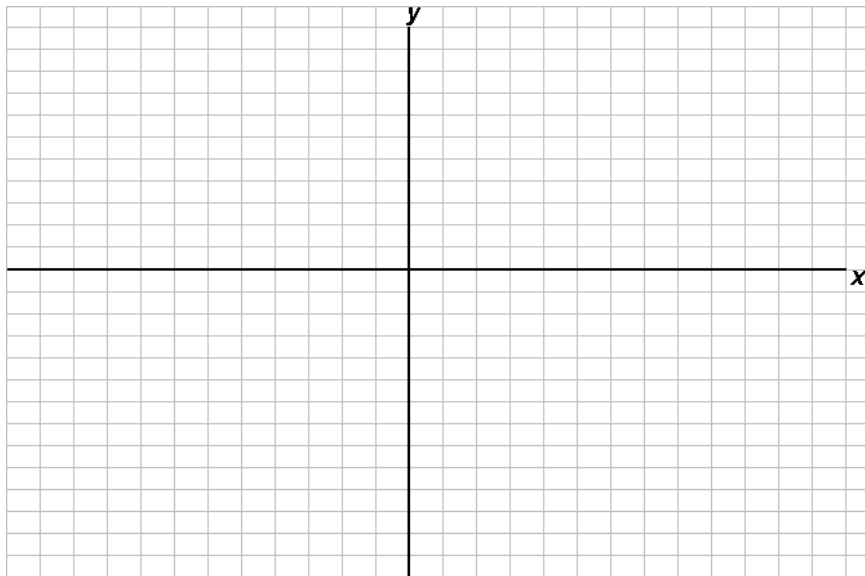
$$f(x) = 8(x - 9)^2 + 5$$

19. What is the maximum product of two numbers whose sum is 22? What numbers yield this product?

20. For the following quadratic function, (a) find the vertex and the axis of symmetry and (b) graph the function. $f(x) = -2x^2 - 4x - 6$



21. For the following equation whose graph is a parabola, (a) find the vertex and the axis of symmetry and (b) graph. $x = 3y^2 + 6y + 5$



22. Solve the inequalities.

a) $x^2 - 4x > 5$

b) $(x-1)(x+2)(2x-1) \leq 0$

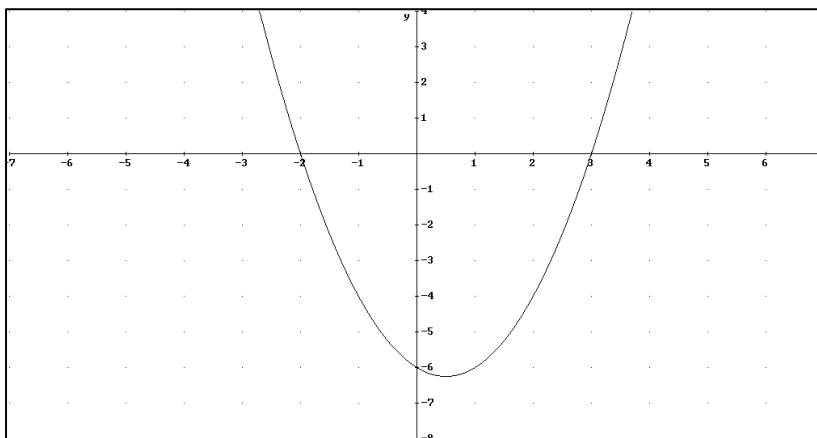
c) $\frac{x+3}{x-4} \geq 0$

23. The graph of a quadratic function f is given. Use the graph to find the solution set of each equation or inequality.

a) $x^2 - x - 6 = 0$

b) $x^2 - x - 6 > 0$

c) $x^2 - x - 6 < 0$



$f(x) = x^2 - x - 6$

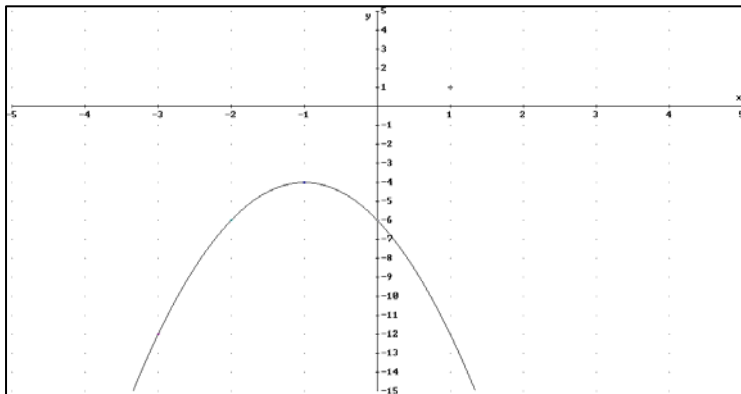
Answers

- 1) a) $x = \pm 2\sqrt{6}$; b) $z = \pm 2\sqrt{10}$; c) $m = 7 \pm \frac{\sqrt{6}}{2}i$
- 2) a) $x = \frac{3}{2} \pm \frac{\sqrt{19}}{2}i$; b) $x = -3\frac{1}{2}$ or $x = \frac{1}{2}$; c) $x = \frac{11 \pm \sqrt{113}}{4}$
- 3) a) $\frac{-1 \pm \sqrt{2}}{2}$; b) $1 \pm \sqrt{3}i$
- 4) a) two real (irrational) solutions; b) two imaginary solutions (complex conjugates)
- 5) $a = 9$
- 6) $x = \frac{1}{15}$ or $x = 2$
- 7) a) $x = -\frac{3}{2}$; b) $x = 27$ or $x = -18\frac{26}{27}$
- 8) One pipe takes 3 hours and the other pipe takes 6 hours to fill the tank.
- 9) It takes James $\frac{5}{2} + \frac{\sqrt{37}}{2} \approx 5.54$ hours to paint the fence, and it takes Kory $\frac{7}{2} + \frac{\sqrt{37}}{2} \approx 6.54$ hours to point the fence.
- 10) The speed of the plane is 255 mph.
- 11) The speed of Jim's canoe in still water is $1 + \sqrt{5} \approx 3.24$ mph.
- 12) The legs are 7 feet and 24 feet.
- 13) $t = \frac{3a \pm \sqrt{9a^2 - 2(3+B)}}{2}$
- 14) $h = \pm d^2 \sqrt{\frac{k}{l}}$
- 15) $g = \frac{kl}{p^2}$
- 16) The dimensions of the original rectangle are 7 meters and 12 meters.
- 17) The rocket will be 550 ft from the ground at $\frac{26 - 3\sqrt{14}}{4} \approx 3.69$ sec
and $\frac{26 + 3\sqrt{14}}{4} \approx 9.31$ sec.

- 18) The vertex is $(9, 5)$, the axis of symmetry is $x = 9$, the minimum value is 5 and the range is $[5, \infty)$. The graph is narrower than the graph of $y = x^2$.
- 19) The maximum product is 121 and the two numbers are 11 and 11.
- 20) The vertex is $(-1, -4)$ and the axis of symmetry is $x = -1$. Five points on the parabola are $(-3, -12)$, $(-2, -6)$, $(-1, -4)$, $(0, -6)$, $(1, -12)$.
- 21) The vertex is $(2, -1)$ and the axis of symmetry is $y = -1$. Five points on the parabola are $(14, 1)$, $(5, 0)$, $(2, -1)$, $(5, -2)$, $(14, -3)$.
- 22) a) $(-\infty, -1) \cup (5, \infty)$; b) $(-\infty, -2] \cup [\frac{1}{2}, 1]$; c) $(-\infty, -3] \cup (4, \infty)$
- 23) a) $x = -2$ or $x = 3$; b) $(-\infty, -2) \cup (3, \infty)$; c) $(-2, 3)$

Graphs for 20 and 21:

20)



21)

