18 Solving Quadratic Equations by Factoring (5.5)

Quadratic equations are <u>equations</u> (not just polynomials) of the second-degree, that is, they have an $\mathbf{x^2}$ term in them**.** Examples are: $x^2 + x = 5$ and $x^2 - 2x + 35 = 0$.

The standard form of a quadratic equation is: $\mathbf{ax^2 + bx + c = 0.}$

To solve the quadratic equation $\mathbf{x^2 + 7x + 6 = 0}$, we must find the values of **x** that make this equation true. We can do this by <u>factoring</u>.

Factoring a polynomial gives us a <u>product</u> of factors:

$$x^2 + 7x + 6 = (x + 6)(x + 1)$$

If a <u>product</u> of two numbers equals zero, for example, $\mathbf{ab = 0}$, then <u>either</u> **a** must equal zero <u>or</u> **b** must equal zero (because anything multiplied by zero equals zero), or both may equal zero. This is called the ***principle of zero products***.

So, if $\mathbf{(x + 6)(x + 1) = 0}$, then <u>either</u> $\mathbf{(x + 6) = 0}$ <u>or</u> $\mathbf{(x + 1) = 0}$. To solve the original equation, we solve for x in both of these equations:

$$\begin{array}{rcl} x + 6 = 0 \\ \underline{\;\; -6 \quad -6} \\ x = -6 \end{array} \qquad\qquad \begin{array}{rcl} x + 1 = 0 \\ \underline{\;\; -1 \;\; -1} \\ x = -1 \end{array}$$

The <u>solutions</u> for $\mathbf{x^2 + 7x + 6 = 0}$ are **–6** and **–1**.

If the polynomial turns out to be a perfect square, you only get one solution (or really the same solution twice):

$$x^2 + 6x + 9 = 0 \;\rightarrow\; (x + 3)(x + 3) = 0 \;\rightarrow\; (x + 3)^2 = 0$$

$$\begin{array}{l} x + 3 = 0 \\ x = -3 \end{array} \qquad\qquad \begin{array}{l} x + 3 = 0 \\ x = -3 \end{array}$$

The solution is $x = -3$. In this case, <u>both</u> factors will equal zero when $x = -3$.

<u>**Solving a Quadratic Equation**</u>

Step 1: Make sure all terms are on one side of the equation and zero is on the other side.

a. $x^2 - 7x = -12 \;\rightarrow\; x^2 - 7x + 12 = 0$

b. $5x^2 = 15x \;\rightarrow\; 5x^2 - 15x = 0$

c. $x(x - 6) = 27 \;\rightarrow\; x(x - 6) - 27 = 0$

d. $(2x + 5)(x + 9) = 15$
$\rightarrow\; (2x + 5)(x + 9) - 15 = 0$

Step 2: If some terms were written in factored form originally, <u>do all the multiplication and combine like terms</u>.

c. $x(x - 6) - 27 = 0 \rightarrow x^2 - 6x - 27 = 0$

d. $(2x + 5)(x + 9) - 15 = 0$

$\rightarrow 2x^2 + 23x + 45 - 15 = 0$

$\rightarrow 2x^2 + 23x + 30 = 0$

Step 3: Factor and solve by setting each factor equal to zero:

a. $x^2 - 7x + 12 = 0$ $\quad (x - 3)(x - 4) = 0$

$x - 3 = 0 \quad\quad x - 4 = 0$

$x = 3 \quad\quad x = 4$

The solutions are x = 3 or x = 4.

b. $5x^2 - 15x = 0$ $\quad 5x(x - 3) = 0$

$5x = 0 \quad\quad x - 3 = 0$

$x = 0 \quad\quad x = 3$

The solutions are x = 0 or x = 3.

c. $x^2 - 6x - 27 = 0$ $\quad (x - 9)(x + 3) = 0$

$x - 9 = 0 \quad\quad x + 3 = 0$

$x = 9 \quad\quad x = -3$

The solutions are x = 9 or x = -3.

d. $2x^2 + 23x + 30 = 0$ $\quad (2x + 3)(x + 10)$

$2x + 3 = 0 \quad\quad x + 10 = 0$

$2x = -3 \quad\quad x = -10$

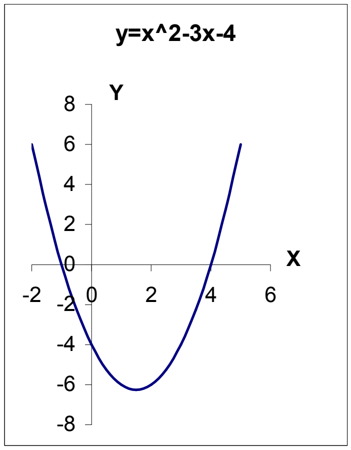
$x = -3/2$

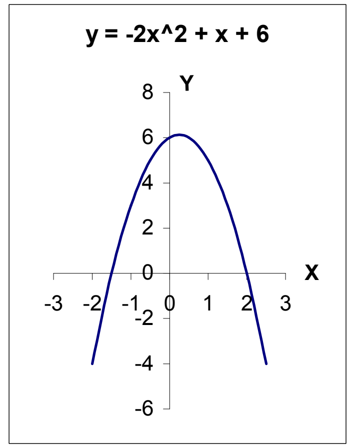
The solutions are x = -3/2 or x = -10.

So why are we doing this?

This section is an introduction to topics in Math 102 – problem solving using quadratics and graphing quadratic functions. Quadratic functions are useful when we are trying to find the maximum or minimum of a function, such as maximizing area or minimizing cost. These topics will be covered later.

The graph of a quadratic function is called a parabola and looks like these:





With what you know now, you can find the **x-intercepts** of these graphs. The x-intercepts are where the graph <u>crosses the x-axis</u>. At these points, **y = 0**. (In Section 3.2, you looked at the y-intercept of straight-line functions - where the line crossed the y-axis. This concept is similar.) When you solve a quadratic equation, you are finding the x-intercepts of the quadratic function (called finding the "zeros" or "roots").

For the graphs above:

$y = x^2 - 3x - 4$ $\qquad\qquad$ $y = -2x^2 + x + 6$

Set y = 0:

$0 = x^2 - 3x - 4$ $\qquad\qquad$ $0 = -2x^2 + x + 6$

Factor and solve:

$0 = (x + 1)(x - 4)$
$x + 1 = 0$ or $x - 4 = 0$
$x = -1$ or $x = 4$

$0 = (-2x - 3)(x - 2)$
$-2x - 3 = 0$ or $x - 2 = 0$
$x = -3/2$ or $x = 2$

Using these values of x, and remembering y = 0, we can state these values as ordered pairs of coordinates on a graph in the form **(x, y).** These represent the points on the graphs which are the x-intercepts:

$x = -1, y = 0$ and $x = 4, y = 0$
x-intercepts: (-1, 0) and (4, 0)

$x = -3/2, y = 0$ and $x = 2, y = 0$
x-intercepts: (-3/2, 0) and (2, 0)

And you can see above, these points are where the graphs cross their x-axes.

Exercises:

Solve, using the principle of zero products:

1. $(x + 3)(x + 11) = 0$

2. $\frac{2}{7}x\,(4x - 3) = 0$

3. $(x - 7)(x + 75)(x - 43) = 0$

4. $0.35x\,(0.1x + 0.4)(0.05x - 15) = 0$

Solve by factoring:

5. $9x^2 - 16 = 0$

6. $x^2 + 7x = 18$

7. $3x^2 - 10x - 8 = 0$

8. $36x^2 + 84x + 49 = 0$

9. $15x^2 + 10x = 0$

10. $(5x + 3)(x - 1) = 13$

Find the x-intercepts:

11. $y = x^2 - 6x + 9$

12. $y = -3x^2 + 5x + 2$

Answers:

1. $x = -3$ or -11
2. $x = 0$ or ¾
3. $x = 7$ or -75 or 43
4. $x = 0$ or -4 or 300
5. $x = 4/3$ or $-4/3$
6. $x = -9$ or 2
7. $x = -2/3$ or 4
8. $x = -7/6$
9. $x = 0$ or $-2/3$
10. $x = -8/5$ or 2
11. (3, 0)
12. (-1/3, 0), (2, 0)

□