RATIONAL EXPRESSIONS (6.1)

A rational *number* is a quotient of two integers, also called a fraction. A rational *expression* is a quotient of polynomials, like $\frac{x+2}{x-5}$ or $\frac{x^2-2x-3}{4x^2-9}$.

Defined/Undefined: A denominator of zero makes a rational expression undefined. If there is a variable in the denominator of a rational expression, the expression will be undefined for any value of that variable that makes the denominator equal zero.

Example 1. For $\frac{x^2+32}{x-15}$ what value of x makes the expression undefined?

Set the denominator equal to zero and solve: $x - 15 = 0$
$x = 15$

Check: $\frac{15^2+32}{15-15} = \frac{257}{0}$ ☺ denominator equals zero

The expression is **undefined** when **x = 15**.

Note that we are only interested in the denominator for determining when the expression is undefined! Ignore the numerator!

Example 2. For $\frac{x-15}{x^2-2x-3}$ what values of x make the expression undefined?

Set the denominator equal to zero: $x^2 - 2x - 3 = 0$
Factor: $(x + 1)(x - 3) = 0$
Set each factor equal to zero and solve: $x + 1 = 0$, $x = -1$; $x - 3 = 0$, $x = 3$

Check: $\frac{-1-15}{(-1)^2-2(-1)-3} = \frac{-16}{1+2-3} = \frac{-16}{0}$ ☺

$\frac{3-15}{(3)^2-2(3)-3} = \frac{-12}{9-6-3} = \frac{-12}{0}$ ☺

The expression is **undefined** when **x = -1** or when **x = 3.**

Simplifying

Simplifying a rational expression means "dividing out" common factors that occur in <u>both the numerator and denominator</u>.

You can simplify a rational expression **if:**

1. The numerator and denominator are both completely factored. Factoring means expressing both the numerator and denominator as **products** of factors **– even if 1 is one of the factors.**

AND

2. There are factors in **both** the numerator and denominator that are **exactly alike**.

If the above is true, we can "cancel" the common factor in the numerator and denominator, since anything divided by itself equals one. Here's how it works.

<u>Example 1.</u> Simplify $\frac{12x^3}{2x}$

Since you've already learned how to do operations with exponents, there are actually two ways you could do a simple problem like this. **Notice that both the numerator and the denominator are already expressed as products in the original expression! This is essential!**

Method 1: Factor completely and cancel. $\frac{2 \bullet 2 \bullet 3 \bullet x \bullet x \bullet x}{2 \bullet x} \rightarrow \frac{\cancel{2} \bullet 2 \bullet 3 \bullet \cancel{x} \bullet x \bullet x}{\cancel{2} \bullet \cancel{x}} = 6x^2$

Method 2: Use the laws of exponents. $\frac{12x^3}{2x} = \frac{12}{2} \bullet \frac{x^3}{x} = 6\left(x^{(3-1)}\right) = 6x^2$

For more complex polynomials, we'll use the factor and cancel method

<u>Example 2.</u> Simplify $\frac{16x^2 - 8x}{4x - 2}$

Factor: $\frac{16x^2 - 8x}{4x - 2} = \frac{(8)(x)(2x-1)}{(2)(2x-1)}$ **Put parentheses around *all* of your factors – it will help you see them as separate entities**.

Now that we have the numerator and denominator expressed as <u>products</u>, we can cancel terms that are exactly alike (they are equal to "1"), and divide 8 by 2.

$$\frac{(8)}{(2)} \bullet \frac{(x)\cancel{(2x-1)}}{\cancel{(2x-1)}} = 4x$$

Sometimes, two factors look *almost* alike, except that the terms in them have opposite signs, such as **(x – 1)** compared to **(1 – x),** or **(– x – 1)** compared to **(x + 1)**. In order to simplify, we must make them look *exactly* alike. How do we do that? By factoring a **–1** out from one of them. Note that all of the terms in one factor must have signs opposite from all the corresponding terms in the other in order to succeed.

That is: $(x^3 + x - 1)$ has opposite signs from $(1 - x - x^3)$ and we can fix that!

BUT $(x^3 + x - 1)$ is not the opposite of $(1 - x + x^3)$ because the x^3 terms have the same sign in both expressions, and we can't make them alike.

Let's make **(1 – x)** look like **(x – 1)**.

Factor a **–1** out of **(1 – x):** $(1 - x) = -1(-1 + x) = -(x - 1)$

Now, if we are trying to simplify $\frac{(x-1)}{(1-x)}$ we would rewrite it as $\frac{(x-1)}{-(x-1)}$ or $-\frac{(x-1)}{(x-1)}$

Note that the negative sign, really a multiplier of –1, is moveable.

And now cancel: $$-\frac{\cancel{(x-1)}}{\cancel{(x-1)}} = -1$$

Example 3. Simplify $\frac{x^2-9}{3-x}$

Factor: $\frac{x^2-9}{3-x} = \frac{(x+3)(x-3)}{(3-x)}$ Put parentheses around all factors!

Factor a –1 out of the (3 – x) term in the denominator:

$$\frac{(x+3)(x-3)}{-(-3+x)} = \frac{(x+3)(x-3)}{-(x-3)} = -\frac{(x+3)(x-3)}{(x-3)}$$

and cancel: $$-\frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}} = -(x+3) \quad or \quad -x-3$$

Problems

For what values of x is the rational expression undefined?

1. $\dfrac{34x-5}{-3x}$

2. $\dfrac{x^2+5}{3-x}$

3. $\dfrac{16-x^2}{x^2-4x-5}$

Simplify (if possible):

4. $\dfrac{16x^3y}{8xy^2}$

5. $\dfrac{7x-14}{5x-10}$

6. $\dfrac{x^2-49}{x^2+8x+7}$

7. $\dfrac{5x-10}{5x}$

8. $\dfrac{x-4}{8-2x}$

9. $\dfrac{x^2+3}{x+3}$

10. $\dfrac{x^2-4x-5}{-x^2+x+2}$

Answers

1. x = 0

2. x = 3

3. x = 5, x = – 1

4. $\dfrac{2x^2}{y}$

5. $\dfrac{7}{5}$

6. $\dfrac{x-7}{x+1}$

7. $\dfrac{x-2}{x}$

8. $-\dfrac{1}{2}$

9. Already simplified: $\dfrac{\left(x^2+3\right)}{\left(x+3\right)}$

10. $-\dfrac{x-5}{x-2}$ *or* $\dfrac{5-x}{x-2}$ *or* $\dfrac{x-5}{2-x}$