

Mat 102/EXL
Quadratic Equations #17

An equation that can be written in the form: $ax^2 + bx + c = 0$ where a, b, and c are real numbers, $a \neq 0$, is a **quadratic equation**. The given form is called **standard form**.

1. Derive the quadratic formula by completing the square on the above equation. We want the students to know how it was developed, rather than just thinking it is “magical”.

2. The solutions of $ax^2 + bx + c = 0$ are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

3. Solve each equation. Write radicals in simplified form.

a) $4x^2 + 5x - 6 = 0$

b) $x^2 = 4x + 7$

c) $x^2 + x = -1$

The Discriminant $b^2 - 4ac$

Nature of Solutions

0

One solution; a rational number

Positive

Two different real-number solutions

Perfect square

Solutions are rational

Not a perfect square

Solutions are irrational conjugates

Negative

Two different imaginary-number solutions (complex conjugates)

Determine the nature of the solutions for each equation.

1. $9x^2 - 12x + 4 = 0$

2. $x^2 + 5x + 8 = 0$

3. $x^2 + 5x + 6 = 0$

Writing Equations from Solutions

Find a quadratic equation whose solutions are given.

1. $x = 3$ and $x = \frac{-2}{5}$

2. $x = \pm 2i$

3. $x = \pm\sqrt{3}$

Equations Quadratic in Form

Solve each equation. Write radicals in simplified form.

1. $x^4 - 9x^2 + 8 = 0$

2. $x - \sqrt{x} = 4$

3. $\left(\frac{x+3}{x-3}\right)^2 - \left(\frac{x+3}{x-3}\right) - 6 = 0$

Tip: For equations to be quadratic in form, one exponent on one of the variables must be double the other.