7 Properties of Equalities and Solving Equations (2.1 - 2.3)

Equation Solving Procedure

1. If necessary, use distributive law to remove parentheses. Then combine like terms (simplify) on both sides of the equals sign if possible.
2. (OPTIONAL) Multiply on both sides to "clear" fractions or decimals if desired.
3. Get all terms with variables on one side and all constant terms on the other side using the addition principle.
4. Combine like terms on both sides if possible.
5. Use the multiplication/division principle to create a coefficient of 1 on the variable.
6. Check your solution by substituting into the original equation if you have time.

Examples

Solve:

$5 = 3 - 4x$

Skip Step 1. (There are no parentheses. The left side is simplified. The right side has no like terms to combine.)

Skip Step 2. (There are no fractions or decimals in the equation.)

$5 = 3 - 4x$
$\underline{-3 \quad -3}$
$2 = -4x$

Step 3. Subtract 3 from both sides so that all variable terms are on the right side and all constant terms are on the left.

Skip step 4. (There are no like terms to combine on either side.)

$\frac{2}{-4} = \frac{\cancel{-4}x}{\cancel{-4}}$

Step 5. Divide both sides by –4 (or multiply both sides by –1/4) to make the coefficient on x equal to 1.

$-\frac{2}{4} = x$

Reduce your fractional answer if possible.

$-\frac{1}{2} = x$

Step 6. Check the solution by substituting into the original equation.

? $5 = 3 - 4\left(-\frac{1}{2}\right)$

? $5 = 3 + \frac{4}{2}$

? $5 = 3 + 2$

? $5 = 5$

True! The solution $x = -\frac{1}{2}$ is correct.

Solve:

$3(y - 6) + 2 = 4(y + 2) - 21$	Remove parentheses (distributive law).
$3y - 18 + 2 = 4y + 8 - 21$	Simplify both sides.
$3y - 16 = 4y - 13$ $-3y \quad\quad -3y$ $-16 = y - 13$	Get variable terms on one side by subtracting 3y from both sides (or subtracting 4y from both sides if you prefer).
$+13 \quad\quad +13$ $-3 = y$	Get constant terms on the other side by adding 13 to both sides. Equation is solved.
? $3(-3 - 6) + 2 = 4(-3 + 2) - 21$? $3(-9) + 2 = 4(-1) - 21$? $-27 + 2 = -4 - 21$	Check by substituting solution in original equation.
$-25 = -25$	True. The solution y = -3 is correct.

Solve:

$\frac{2}{3}\left(x - \frac{1}{2}\right) = \frac{5x}{9} - 3$	Remove parentheses.
$\frac{2x}{3} - \frac{2}{6} = \frac{5x}{9} - 3$	
$18\left(\frac{2x}{3} - \frac{2}{6}\right) = 18\left(\frac{5x}{9} - 3\right)$	Since all denominators will divide into 18 (least common denominator), multiply both sides of equation by 18.
$18 \cdot \frac{2x}{3} - 18 \cdot \frac{2}{6} = 18 \cdot \frac{5x}{9} - 18 \cdot 3$	
$6 \cdot 2x - 3 \cdot 2 = 2 \cdot 5x - 18 \cdot 3$	
$12x - 6 = 10x - 54$ $-10x \quad\quad -10x$	Get variable terms on one side.
$2x - 6 = -54$ $+6 \quad\quad +6$ $2x = -48$	Get constant terms on the other side.
$\frac{\cancel{2}x}{\cancel{2}} = \frac{-48}{2}$ $x = -24$	Multiply (or divide) both sides to create coefficient of 1 on the variable.

Solve:

$0.2x - 0.3 = 0.4x + 0.28$
$100(0.2x - 0.3) = 100(0.4x + 0.28)$
$100(0.2x) - 100(0.3) = 100(0.4x) + 100(0.28)$
$20x - 30 = 40x + 28$

Since the "most" number of decimal places in any term is 2 (hundredths place), multiply both sides by 100 to clear the decimals if desired. (Multiplying a number by 100 will "move" its decimal 2 places to the right.)

$$\begin{array}{rcl} 20x - 30 &=& 40x + 28 \\ \underline{-20x\quad} & & \underline{-20x\quad} \\ -30 &=& 20x + 28 \end{array}$$

Get variables on one side.

$$\begin{array}{rcl} -30 &=& 20x + 28 \\ \underline{-28} & & \underline{\quad -28} \\ -58 &=& 20x \end{array}$$

Get constants on the other side.

$$\frac{-58}{20} = \frac{\cancel{20}x}{\cancel{20}}$$

Multiply or divide to get coefficient of 1 on the variable.

$-2.9 = x$

Since original equation was in decimal form, it would be okay to write solution as a decimal.

NOTE: Two weird things that can happen and what they mean:
What if, when solving your equation, the variable terms cancel each other out?
If you end up with a TRUE equation (for example, 5 = 5) then this means that your equation has an infinite number of solutions (ie. EVERY real number is a solution to this equation).
If you end up with a FALSE equation (for example 3 = 7) then this means that your equation has NO solutions (ie. there is NO real number that will solve this equation).

Example: $\frac{1}{3}(2x+5) = \frac{1}{6}(4x+10)$

$$\frac{2x}{3} + \frac{5}{3} = \frac{4x}{6} + \frac{10}{6}$$

$$\frac{2x}{3} + \frac{5}{3} = \frac{2x}{3} + \frac{5}{3}$$

$$-\frac{2x}{3} \qquad -\frac{2x}{3}$$

$$\frac{5}{3} = \frac{5}{3} \quad \text{always true!}$$

x = any real number

Example: $x - 5 = 3\left(\frac{x}{3} - \frac{5}{12}\right)$

$$x - 5 = \frac{3x}{3} - \frac{15}{12}$$

$$x - 5 = x - \frac{5}{4}$$

$$-x \qquad -x$$

$$-5 = -\frac{5}{4} \quad \text{never true!}$$

no solution

Problems

1. $x - 8 = -10$
2. $x + 7 = 2x - 1$
3. $-x + 2 = -4$
4. $3y = 12$
5. $\frac{2t}{3} = 4$
6 $7z - 5 = 12z + 10$
7. $-2(3a - 4) + a = 3a - 2$
8. $-2x + 1 = 2(-x + \frac{1}{2})$
9. $-\frac{1}{2}(3x - 4) = \frac{2x}{3} - \frac{1}{8}$
10. $\frac{4}{5}\left(x + \frac{1}{2}\right) - \frac{3}{5} = -x$
11. $x + 2 = x - 3$
12. $0.2(25x - 35) + 0.12x = 0.4(21.25 + 0.3x)$
13. $3x = 6x$
14. $\frac{4x}{5} = 2x - 1$

Answers

1. $x = -2$
2. $x = 8$
3. $x = 6$
4. $y = 4$
5. $t = 6$
6. $z = -3$
7. $a = \frac{5}{4}$
8. x = any real number
9. $x = \frac{51}{52}$
10. $x = \frac{1}{9}$
11. no solutions
12. $x = 3.1$
13. $x = 0$ (combine variable terms first!)
14. $x = \frac{5}{6}$