

25 Problem Solving – Interest, Increase/Decrease, and Denominations of Money (2.7)

“Interest” is what a bank pays you for depositing your money with them. Problems involving **simple interest** for **one year** use a simple formula:

$$\text{Interest} = (\text{principal})(\text{rate}) \quad \text{or} \quad I = pr$$

where principal means the *amount invested* and rate is a percent rate *converted to a decimal*.

So, if you put \$100 in a savings account that pays you 5% simple interest, then after 1 year your money will have earned:

$$(\$100)(.05) = \$5 \text{ interest.} \quad \underline{\text{Remember to convert the rate to a decimal!}}$$

If you take a total investment amount and split it between two different accounts with different rates, it’s only a little more complicated. Let’s say you want to invest \$8000 in two accounts, one paying 7% interest and one paying 9% interest, and you want to make a total of \$650 in interest in one year. How much should you put into each account?

First, let’s get the data organized with a table:

Put the rates in decimal form.

(Note that rates do not add.)

	P	•	R =	I
#1			.07	
#2			.09	
Total	8000		NA	650

How do we write the two amounts? Pick one and call it x . How much is the other one? If the total is 8000, then the other one must be $8000 - x$.

	P	•	R =	I
#1	x		.07	
#2	$8000 - x$.09	
Total	8000		NA	650

Multiply principal times rate to get interest amounts for each investment. The total interest is 650, so the two interest amounts add up to equal 650 – that’s the equation to solve:

$$.07x + .09(8000 - x) = 650$$

Using the Decimals

$$.07x + .09(8000 - x) = 650$$

1. Clear the parentheses:

$$.07x + 720 - .09x = 650$$

	P	•	R =	I
#1	x		.07	$.07x$
#2	$8000 - x$.09	$.09(8000 - x)$
Total	8000		NA	650

Clearing the Decimals

$$.07x + .09(8000 - x) = 650$$

1. Clear the decimals – multiply both sides by 100.

$$7x + 9(8000 - x) = 65000$$

Then clear the parentheses:

$$7x + 72000 - 9x = 65000$$

2. Simplify the left hand side:

$$720 - .02x = 650$$

3. Get the x term by itself on one side:

$$\begin{array}{r} 720 - .02x = 650 \\ - 720 \quad - 720 \\ \hline - .02x = - 70 \end{array}$$

4. Divide both sides to solve for x:

$$\frac{-.02x}{-.02} = \frac{-70}{-.02}$$

$$x = 3500$$

2. Simplify the left hand side:

$$72000 - 2x = 65000$$

3. Get the x term by itself on one side:

$$\begin{array}{r} 72000 - 2x = 65000 \\ - 72000 \quad - 72000 \\ \hline - 2x = - 7000 \end{array}$$

4. Divide both sides to solve for x:

$$\frac{-2x}{-2} = \frac{-7000}{-2}$$

$$x = 3500$$

The amount to invest at 7% is \$3500. The amount to invest at 9% was defined as $8000 - x$, so it would be: $\$8000 - \$3500 = \$4500$.

Example 2 Joe invests some money at 4% and \$3000 more than twice that amount at 5%. After one year, he earns \$430 interest. How much did he invest at each interest rate?

Assign x to the amount invested at 4%.

Translate “\$3000 more than twice that amount” into “ $2x + 3000$ ”.

The two interest amounts add to equal \$430:

$$.04x + .05(2x + 3000) = 430$$

	P	R	I
#1	x	$.04$	$.04x$
#2	$2x + 3000$	$.05$	$.05(2x + 3000)$
Total		<i>NA</i>	430

1. Clear the parentheses:

$$.04x + .10x + 150 = 430$$

2. Add like terms:

$$.14x + 150 = 430$$

3. Subtract 150 from both sides:

$$.14x = 430 - 150 = 280$$

4. Divide both sides by .14:

$$\frac{.14x}{.14} = \frac{280}{.14}$$

$$x = \$2000 \quad \text{this is the amount invested at 4\%}$$

5. Find the other amount by using $2x + 3000$:

$$2(2000) + 3000 = 4000 + 3000 = \$7000 \text{ invested at 5\%}$$

Increase/Decrease Problems

When you buy something at the store, sales tax is added onto the price of the item. The amount of tax is calculated by multiplying the original price by the tax rate. The total cost then is the original price plus the amount of tax – this is a type of *increase* problem. For example, if a shirt is priced at \$15.00 and the sales tax is 6%, then:

$$\text{amount of tax} = .06(15.00)$$

$$\text{and the total cost is: } 15.00 + .06(15.00) = 15.00 + .90 = \$15.90$$

Example If you paid \$46.11 for hardware supplies and sales tax is 6%, what was the cost of the supplies before tax?

$$\text{Let } x \text{ be the original cost and add 6\% of } x: \quad x + .06x = 46.11$$

The two x terms on the left side of the equation are “like terms” and can be added. To add them, remember that we can always put a multiplier of “1” in front of anything.

$$x + .06x = 1x + .06x = 1.06x \quad \rightarrow \quad 1.06x = 46.11$$

$$\text{To solve for } x, \text{ divide both sides by } 1.06: \quad \frac{1.06x}{1.06} = \frac{46.11}{1.06} \quad \rightarrow \quad x = \$43.50$$

Decrease problems work the same way, except that the amount of decrease is *subtracted* from the original amount. For example, if the \$15.00 shirt is on sale at “25% off”, then 25% of the original price will be subtracted:

$$15.00 - .25(15.00) = 15.00 - 3.75 = \$11.25 \text{ sale price}$$

Example After a 35% reduction, a bicycle is on sale at \$187.85. What was the original price of the bike?

Let x be the original price and *subtract* 35% of x :

$$x - .35x = 187.85$$

Remember the “1” in front of the first x term!

$$1x - .35x = 187.85$$

Combine like terms:

$$.65x = 187.85$$

Divide both sides by .65:

$$\frac{.65x}{.65} = \frac{187.85}{.65} \quad \rightarrow \quad x = \$289, \text{ the original price.}$$

Denominations of Money

Problems involving coins or bills with different monetary values, like quarters and dimes, are similar to interest problems. However, instead of applying interest rates, you apply the value of each coin or bill to the total number of items to get a total value. For example, if you have 3 five dollar bills and 4 one dollar bills in your pocket, then the total amount of money you have is:

$$3(\$5) + 4(\$1) = \$15 + \$4 = \$19$$

If you have 3 dimes, 5 nickels, and 6 quarters, then the total value of the money would be:

$$3(\$0.10) + 5(\$0.05) + 6(\$0.25) = \$0.30 + \$0.25 + \$1.50 = \$2.05$$

Example A bag of dimes and quarters has a total value of \$3.95. There are 6 more quarters than dimes. How many are there of each type of coin?

Let x be the number of dimes. Then the number of quarters is $x + 6$. Each dime is worth \$0.10, so the total value of the dimes is $0.10x$. Similarly, the value of the quarters is \$0.25 times the number of quarters. The two values add up to equal the total value of the money: \$3.95:

Number of Coins •	Denomination =	Total Value
x	.10	$.10x$
$x + 6$.25	$.25(x + 6)$
		3.95

$$.10x + .25(x + 6) = 3.95$$

1. Clear the parentheses: $.10x + .25x + 1.5 = 3.95$
2. Combine like terms: $.35x + 1.5 = 3.95$
3. Subtract 1.5 from both sides: $.35x = 3.95 - 1.5 = 2.45$
4. Divide both sides by .35: $\frac{.35x}{.35} = \frac{2.45}{.35} \rightarrow x = 7, x + 6 = 13$

There are 7 dimes and 13 quarters in the bag.

Example After an arts and crafts sale, the cashier has \$360 in fives, tens, and twenties. There are twice as many tens as fives, and there are five more twenties than tens. How many of each type of bill are there?

Let x be the number of fives. The number of tens is *twice as many*, or $2x$. The number of twenties is *five more than the number of tens*, or $2x + 5$. The three values add up to \$360:

$$5x + 10(2x) + 20(2x + 5) = 360$$

Number of Bills •	Denomination =	Total Value
x	5	$5x$
$2x$	10	$10(2x)$
$2x + 5$	20	$20(2x + 5)$
		360

1. Clear the parentheses: $5x + 20x + 40x + 100 = 360$
2. Combine like terms: $65x + 100 = 360$
3. Subtract 100 from both sides: $65x = 360 - 100 = 260$
4. Divide both sides by 65: $\frac{65x}{65} = \frac{260}{65} \rightarrow x = 4, 2x = 8, 2x + 5 = 13$

There are 4 fives, 8 tens, and 13 twenties.

Problems

1. Jill deposited part of \$10,000 in an account at 5% interest and the rest in an account at 3% interest. After one year she had earned \$420 in interest. How much was deposited in each account?
2. Brett invested some money at 4% and \$3000 less than that at 2%. The two investments produced a total of \$240 in one year. How much was invested at each rate?
3. A store sells wheelbarrows for \$40.95, which includes a 5% markup (increase) from the wholesale price. How much does the store pay for a wheelbarrow (before the markup)?
4. A hot dog cost \$2.75 and a bag of chips cost \$0.75. If you want four more hot dogs than bags of chips, how many of each can you buy with \$25.00?
5. After a 10% reduction, a lawnmower is on sale for \$216. What was the original price of the lawnmower?
6. A cashier has 55 bills in fives and tens. The total value is \$375. How many of each denomination does he have?
7. A bag contains nickels and dimes. The total value of the coins is \$1.45. The number of dimes is three less than twice the number of nickels. How many of each type of coin are there?

Answers

1. \$6,000 at 5% and \$4,000 at 3%
2. \$5000 at 4% and \$2000 at 2%
3. \$39
4. 8 hot dogs and 4 bags of chips
5. \$240
6. 35 fives and 20 tens
7. 7 nickels and 11 dimes