

Addition, Subtraction, Multiplication, and Division of Real Numbers (1.5, 1.6)

Rules For Addition of Real Numbers

Examples

Positive Numbers: Add as usual. The answer is positive.

$$8 + 3 = 11$$

Negative Numbers: Add absolute values and make the answer negative.

$$(-8) + (-3) = -11$$

8 + 3 (absolute values of -8 and -3) is 11.
Make answer negative.

A Positive and a Negative Number: Subtract absolute values. Give answer the sign of the number with the largest absolute value.

$$(-8) + 3 = -5$$

Find difference of 8 and 3 (absolute values of - 8 and 3). Difference is 5.
Give answer the sign of the number with the largest absolute value. ($8 > 3$ so answer should have same sign as the -8 (negative answer).)

$$8 + (-3) = 5$$

Find difference of 8 and 3. Difference is 5. Give answer the sign of the number with the largest absolute value.
($8 > 3$ so answer should have same sign as 8 (positive).)

NOTE: If difference is zero, you're done. (Zero is neither positive nor negative.)

One number is zero: Answer is just the other number.

$$8 + 0 = 8$$

$$0 + (-8) = -8$$

$$0 + 0 = 0$$

Collecting Like Terms

When 2 terms have variable factors that are exactly the same, the terms are called **like** or **similar** terms. The distributive law allows us to add (or subtract) like terms by adding (or subtracting) their numerical coefficients. (Answer has same variable factors.)

Examples

7ab and 3ad

are NOT like terms. (ab is not the same as ad)

7ab and 3ba

ARE like terms. (ab is the same as ba: order does not matter in a multiplication)

$7x^2y$ and $3xy^2$

are NOT like terms. (x^2y is not the same as xy^2)

$7x^2y$ and $3x^2$

are NOT like terms. (x^2y is not the same as x^2)

$7x^2y$ and $3yx^2$

ARE like terms. (x^2y is the same as yx^2 : order does not matter in a multiplication)

7x and 3

are NOT like terms. (7x has a variable factor x and 3 does not have a variable factor at all)

$7xy + 3xy = 10xy$ $7xy$ and $3xy$ are LIKE terms. Add their numerical coefficients ($7 + 3$).
 Answer has same variable factors.

$-7xy + (-3xy) = -10xy$
 $-7xy + 3xy = -4xy$
 $7xy + (-3xy) = 4xy$

$7xy + 3x = 7xy + 3x$ $7xy$ and $3x$ are NOT like terms. This expression cannot be simplified further.

Problem Solving

$$\text{First Change} + \text{Second Change} + \text{Third Change} + \dots = \text{Total Change}$$

Remember, “change” could be positive or negative!

Example

I deposited \$200 in my checking account. I then wrote a check for \$50.21 and then I got \$100 cash from the ATM. Then I deposited a check for \$107.34. Then I wrote a check for \$22.81 and another check for \$78.23. What was the total change in my checking account balance?

(200)	+	(-50.21)	+	(-100)	+	(107.34)	+	(-22.81)	+	(-78.23)	=	56.09
deposit		withdrawal		withdrawal		deposit		withdrawal		withdrawal		total change

My balance went UP by \$56.09.

Subtraction of Real Numbers

$$a - b = a + (-b)$$

To subtract two numbers a and b , number
 change the operation to addition and change the sign on the number being subtracted (b). Then follow the rules for addition.

Examples

$$\begin{aligned}
 5 - 3 &= 5 - 3 \\
 &\downarrow \qquad \qquad \text{change sign on} \\
 &= 5 + -3 \leftarrow \text{being subtracted} \\
 &\quad \uparrow \\
 &\text{change subtraction to addition} \\
 &= 5 + (-3) \\
 &= 2 \quad (\text{by addition rules})
 \end{aligned}$$

$$\begin{aligned}
 -5 - (-3) &= -5 - (-3) \\
 &\downarrow \qquad \qquad \text{change sign on number} \\
 &= -5 + (+3) \leftarrow \text{being subtracted} \\
 &\quad \uparrow \\
 &\text{change subtraction to addition} \\
 &= -5 + 3 \\
 &= -2 \quad (\text{by addition rules})
 \end{aligned}$$

$$\begin{aligned}
 -5 - 3 &= -5 - 3 \\
 &\text{leave first number alone} \\
 &\quad \downarrow \quad \text{change sign on number} \\
 &= -5 + (-3) \leftarrow \text{being subtracted} \\
 &\quad \uparrow \\
 &\text{change subtraction to addition} \\
 &= -8 \quad (\text{by addition rules})
 \end{aligned}$$

Multiplication and Division of Real Numbers

Examples

Zero times any number is zero.

$$-1.23(0) = 0$$

Any number (except zero) divided by zero is **undefined**.

$$\frac{7}{0} \text{ is undefined}$$

Zero divided by any number (except zero) is zero.

$$\frac{0}{7} = 0$$

Zero divided by zero is said to be **indeterminate**.

$$\frac{0}{0} \text{ is indeterminate}$$

The product or quotient of 2 positive numbers is positive.

$$7(3) = 21 \quad \frac{10}{5} = 2$$

The product or quotient of 2 negative numbers is also positive.

$$-7(-3) = 21 \quad \frac{-10}{-5} = 2$$

2

The product or quotient of 1 negative and 1 positive number is negative.

$$(-7)(3) = -21 \quad \frac{10}{-5} = -2$$

2

NOTE: When writing a negative fraction such as “negative three fourths”, all three of these expressions are equivalent:

$$-\frac{3}{4} = \frac{-3}{4} = \frac{3}{-4}$$

PROBLEMS:

Perform the following operations, if possible.

- | | | | |
|----------------------------------|---|--------------------|---|
| 1. $-8 + 7$ | 7. $12 + 13$ | 13. $4(-3)$ | 19. $\frac{-1}{2} + \frac{-2}{3}$ |
| 2. $0\left(\frac{15}{19}\right)$ | 8. $\frac{1}{2} \div \left(\frac{-3}{4}\right)$ | 14. $-7x^2 + 3x$ | 20. $8 + (-7)$ |
| 3. $0 - (-4)$ | 9. $\frac{-1}{2} - \frac{2}{3}$ | 15. $-8 + 8$ | 21. $8x^2 - 6x^2$ |
| 4. $1.8 - 7.3$ | 10. $-7 - (-9) + 4 - 3$ | 16. $-5 - (-2)$ | 22. $(-1)(-9)$ |
| 5. $\frac{0}{0}$ | 11. $\frac{10}{0}$ | 17. $12 - 15$ | 23. $-7xy + 3x - 8x + 4yx - 5x^2$ |
| 6. $-4xy - 6xy$ | 12. $-5 + (-10)$ | 18. $\frac{0}{10}$ | 24. $\left(\frac{2}{3}\right)\left(\frac{-6}{7}\right)$ |

25. At 7:00 A.M. the temperature was 58 degrees. The temperature rose 5 degrees, then fell 3 degrees, then rose 2 degrees, then fell 4 degrees, then rose 10 degrees all before noon. What was the temperature at noon?

ANSWERS:

- | | | |
|--------------------|--|--------------------|
| 1. -1 | 8. $-\frac{2}{3}$ | 15. 0 |
| 2. 0 | 9. $-\frac{7}{6}$ | 16. -3 |
| 3. 4 | 10. 3 | 17. -3 |
| 4. -5.5 | 11. undefined | 18. 0 |
| 5. indeterminate | 12. -15 | 19. $-\frac{7}{6}$ |
| 6. $-10xy$ | 13. -12 | 20. 1 |
| 7. 25 | 14. cannot be simplified further | 21. $2x^2$ |
| 22. 9 | 23. $-3xy - 5x - 5x^2$ or $-3yx - 5x - 5x^2$ (terms can be in any order) | |
| 24. $-\frac{4}{7}$ | 25. 68 degrees | |