14 Multiplying Polynomials and Special Products (4.5, 4.6)

Monomial x Monomial

Example 1: Multiply the monomial $-2x^2$ by the monomial $6xy$.

$(-2x^2) \cdot (6xy)$
$= (-2)(x^2)(6)(x)(y)$
$= (-2)(6)(x^2)(x)(y)$
$= (-12)(x^2 x^1)(y)$
$= (-12)(x^3)(y)$
$= -12x^3y$

Since $-2x^2 = (-2)(x^2)$ and since $6xy = (6)(x)(y)$ we can think of this product as just a string of multiplications.
We can rearrange using commutative property of multiplication and do what multiplications we can (using exponent properties where appropriate).

Monomial x Polynomial

Suppose you want to multiply a monomial by a polynomial with more than one term.
Example 2: Multiply the monomial $3xy^2$ by the trinomial $xy + 4x^2 - y^3$.

$(3xy^2)(xy + 4x^2 - y^3)$
$= (3xy^2)(xy) + (3xy^2)(4x^2) - (3xy^2)(y^3)$
$= 3x^2y^3 + 12x^3y^2 - 3xy^5$

Use the distributive law ($a(b+c) = ab + ac$) and multiply every term in the second polynomial by the monomial.
Do the resulting multiplications as in Example 1.
If there are any like terms, combine them.

Polynomial x Polynomial

Suppose you want to multiply a polynomial with more than one term by another polynomial with more than one term. Simply multiply every term of the first polynomial by every term of the second polynomial and add the resulting multiplications.

Example 3: Multiply $2x^3 - x + 1$ by $-x^2 - x - 8$

$(2x^3 - x + 1)(-x^2 - x - 8)$
$= 2x^3(-x^2) + 2x^3(-x) + 2x^3(-8)$
$\quad + -x(-x^2) + -x(-x) + -x(-8)$
$\quad + 1(-x^2) + 1(-x) + 1(-8)$
$= -2x^5 \quad -2x^4 \quad -16x^3$
$\quad + x^3 \quad + x^2 \quad + 8x$
$\quad - x^2 \quad - x \quad - 8$
$= -2x^5 - 2x^4 - 15x^3 + 7x - 8$

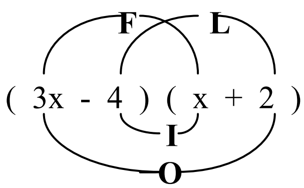
Start with the first term of the first polynomial and multiply it by every term of the second polynomial and add these products. Then take the second term of the first polynomial and multiply it by every term of the second polynomial, etc. Add the resulting multiplications.

Combine like terms and write your answer in descending powers.

Special Products

Although a **binomial times a binomial** can be done as in Example 3 above, they occur so often in algebra that we have a little memory device called **FOIL** to help us multiply them quickly. Learning this method will help you greatly in the next chapter on factoring.

Example 4: Multiply the binomial $3x - 4$ by the binomial $x + 2$.



FOIL stands for "**First, Outer, Inner, Last**". Multiply "First" terms in each binomial, then "Outer" terms, then "Inner" terms, then "Last" terms.

	F	O	I	L	
=	$(3x)(x)$ +	$(3x)(2)$ +	$(-4)(x)$ +	$(-4)(2)$	
=	$3x^2$ +	$6x$	$-4x$	-8	Combine like terms and write your answer in descending powers.
=	$3x^2 + 2x - 8$				

There are two special cases of a binomial times a binomial that you should take extra note of. Being able to recognize these two special cases when you see them will save you a little time in multiplying. Also, these two special cases are important in chapters to come.

(SUM)*(DIFFERENCE)

Suppose you are multiplying 2 binomials that are identical in every way except that one is a sum and the other is a difference. Watch what happens when we use the FOIL method to multiply:

Example 5

$(3x + 2)(3x - 2)$

	F	O	I	L
=	$(3x)(3x)$ +	$(3x)(-2)$ +	$(2)(3x)$ +	$(2)(-2)$
=	$9x^2$	$-6x$	$+6x$	-4
=	$9x^2 - 4$			

Notice that these binomials are the same except that one is a sum and the other is a difference.
The "first" product will be 3x times itself (or $(3x)^2$). The "outer" and "inner" multiplications will be the same but they will have opposite signs and will always cancel each other out when we add!
The "last" multiplication will be 2 times itself (or 2^2) and will always turn out negative because of the difference in signs.

The result of multiplying a sum and difference is called a **difference of squares** ("difference" because we always end up with a minus sign between the 2 terms and "squares" because both terms result from squaring something). Use this knowledge to save yourself time!

Example 6

Multiply:

$(2y - x)(2y + x)$
$= 4y^2 - x^2$

Recognize that this is a sum times a difference. Square the 2y, write down a minus sign, and square the x. You're done!

(BINOMIAL) ^{2}

The second special case involving binomial multiplication is referred to as the **"square of a binomial"**. There is a pattern to be observed here also that may save you a little time in multiplying.

Example 7

$(2x - 3)^2$
$= (2x - 3)(2x - 3)$
F O I L
$= (2x)(2x) + (2x)(-3) + (-3)(2x) + (-3)(-3)$
$= 4x^2 \quad - 6x \quad - 6x \quad + 9$

$= 4x^2 \quad - 12x \quad + 9$

To "square" something is to multiply it by itself. Apply the FOIL method and think about what is happening. The "first" multiplication is 2x times itself (or $(2x)^2$). The "outer" and "inner" multiplications are identical and the "last" multiplication is 3 times itself (or 3^2). The outer and inner multiplications will be either both positive or both negative (depending on the signs in the original binomial). The last multiplication will always be positive (because it will be either a positive times a positive or a negative times a negative). Use this knowledge to save a little time!

Example 8

$(-y + 7)^2$
$= (-y)^2 + 2(-y)(7) + (7)^2$
$= y^2 \quad - 14y \quad + 49$

Recognize this as the "square of a binomial". For the first multiplication, square the -y. Next, multiply the -y and the +7 to get -7y and then DOUBLE the product ($2(-7y) = -14y$). (Remember, there will be two identical terms for the inner and outer multiplications.) For your last multiplication, square the 7. This last term will always be positive. You are done!

Very important note!!!!! Do not make this all too common mistake: $(a + b)^2 = a^2 + b^2$
This is not true! As we saw in examples 7 and 8, squaring a binomial always results in 3 terms!

Problems

Multiply, simplify, and write your answers in descending order.

1) $(2xy)(-3xy^3)$

2) $x^2(x^3 + 2x^2 - 5)$

3) $-3y(-y^3 - 2x + 1)$

4) $(6x^5)(2x^4)$

5) $(x + 2)(x - 1)$

6) $(x + 2)(x - 2)$

7) $(3x^2 - x + 2)(x - 2x^2)$

8) $(y^2 + 2y + 3)(y^2 - 2y - 3)$

9) $(8x^3 + 1)(-x^3 - 3)$

10) $(4x + 1)(4x - 1)$

11) $(3x + 2)^2$

12) $(y + \frac{1}{2})(y - \frac{1}{2})$

13) $(y - \frac{1}{2})^2$

14) $(xy^2 - 4z)(xy^2 + 4z)$

15) $(4x + y)(x^2 - x - y)$

Answers:

1) $-6x^2y^4$

2) $x^5 + 2x^4 - 5x^2$

3) $3y^4 + 6xy - 3y$

4) $12x^9$

5) $x^2 + x - 2$

6) $x^2 - 4$

7) $-6x^4 + 5x^3 - 5x^2 + 2x$

8) $y^4 - 4y^2 - 12y - 9$

9) $-8x^6 - 25x^3 - 3$

10) $16x^2 - 1$

11) $9x^2 + 12x + 4$

12) $y^2 - \frac{1}{4}$

13) $y^2 - y + \frac{1}{4}$

14) $x^2y^4 - 16z^2$

15) $4x^3 + x^2y - 4x^2 - 5xy - y^2$