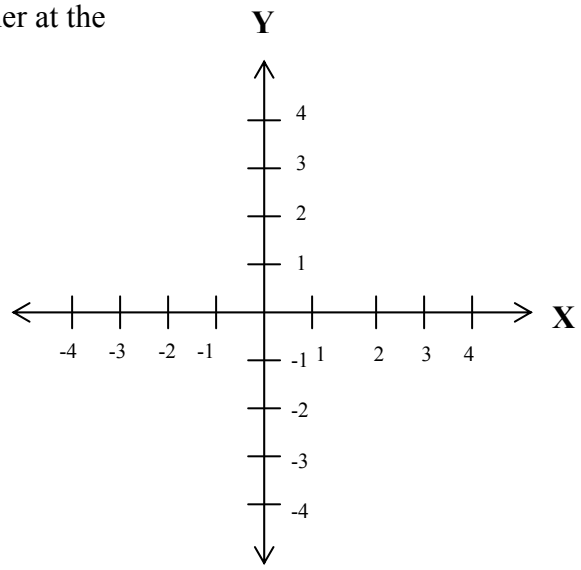
11 LINEAR EQUATIONS AND THEIR GRAPHS (3.1, 3.2)

Coordinate System and Plotting Points

To make a **rectangular coordinate system** (also called a **Cartesian coordinate system**), draw two number lines at right angles intersecting each other at the zero points like so:

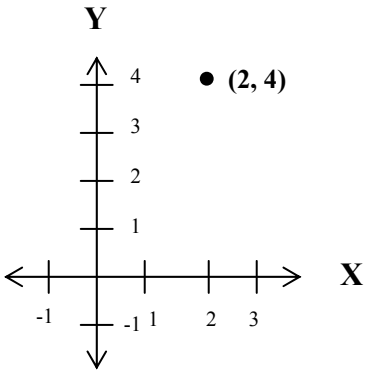


The horizontal number line will be called the **x-axis** and the vertical number line will be called the **y-axis**. The point where the two number lines meet (0 on both number lines) will be called the **origin**. We can use this coordinate system to describe the location of <u>any</u> point on our two-dimensional surface. We will use an **ordered pair** to describe a point's location. Start at the origin. The first member of the ordered pair will tell us how many units to move in the horizontal direction. The second member of the ordered pair will tell us how many units to move in the vertical direction. We will separate these two numbers with a comma and enclose them in parentheses.

Examples

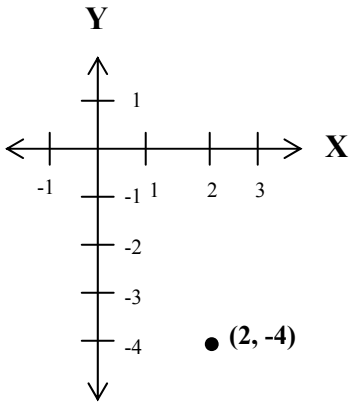
1) Locate the point (2, 4) on a rectangular coordinate system and plot it.

Start at the origin.
Move 2 units to the <u>right</u> and 4 units <u>up</u>.



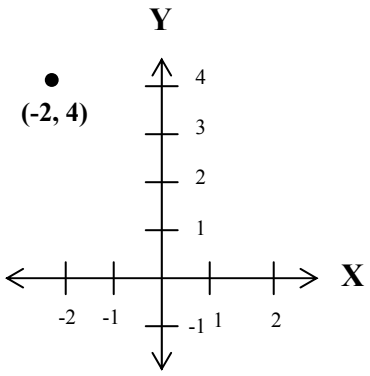
2) Plot the point (2, -4)

Start at the origin.
Move 2 units to the <u>right</u> and 4 units <u>down</u>.



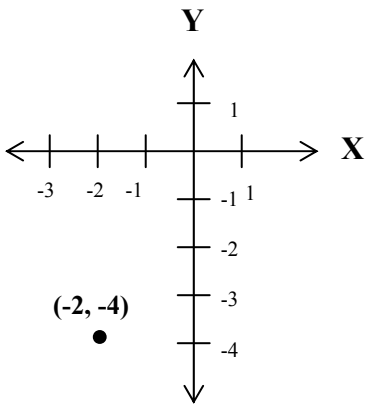
3) Plot the point (-2, 4)

Start at the origin.
Move 2 units left and 4 units up.



4) Plot the point (-2, -4)

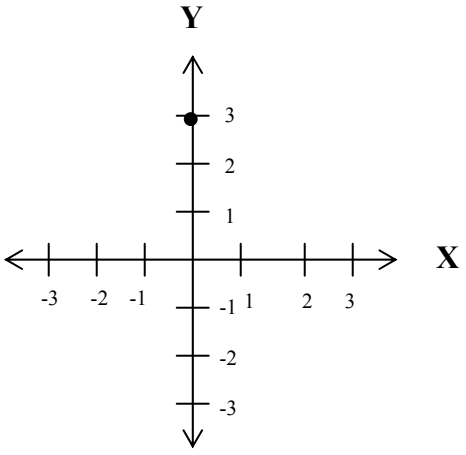
Start at the origin.
Move 2 units left and 4 units down.



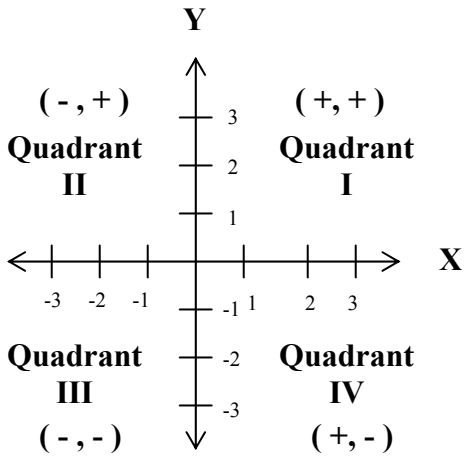
Ordered pairs are also called **coordinates**. The first member is the **x-coordinate** and the second member is the **y-coordinate**.

Find the coordinates of the point shown.

Start at the origin. How far should you move horizontally to get to this point? You don't need to move horizontally at all! So the x-coordinate of the point is 0. How far should you move vertically? Up 3 units. So the y-coordinate of the point is 3. The ordered pair (or coordinates) that describe the location of this point is **(0, 3)**.



Notice that a coordinate system "divides" the paper into 4 areas. These areas have names, **Quadrants I, II, III, and IV**. We can say a point is "in Quadrant II", etc. If a point lies directly on one of the axes, it does NOT lie in any of the quadrants. Notice that all the points in Quadrant I will have a positive x-coordinate and positive y-coordinate. All points lying in Quadrant II will have negative x-coordinates and positive y-coordinates. All points lying in Quadrant III will have negative x-coordinates and negative y-coordinates. All points lying in Quadrant IV will have positive x-coordinates and negative y-coordinates. If either one of the coordinates of a point is 0, the point will lie on one of the axes.



Graphing Equations

Consider the equation $x + y = 6$. Since the equation has 2 variables, a "solution" for this equation would require a value for x and a value for y. If x=2 and y=4, the equation would be true. So x=2 and y=4 is a solution to this equation. We will use ordered pairs to write solutions to equations with 2 variables. The first member of the ordered pair will be the value for x and the second member will be the value for y. So (2, 4) is a solution to the above equation. But there are many others! Some other solutions are (1, 5), (0, 6), (3, 3), (6, 0), and (-6, 12). In fact, there are an infinite number of solutions to this equation. We can represent all the solutions of this equation on a rectangular coordinate system. If you were to plot some of the ordered pair solutions to this equation, you would see that they appear to lie in a straight line. By drawing a straight line through the plotted points, with an arrow on each end (to show that the line goes on indefinitely), we are "graphing" the equation $x + y = 6$. The coordinates of any point on that line will make the equation true.

Not all equations have graphs that are straight lines. But any equations of the form $\mathbf{y = mx}$ or $\mathbf{y = mx + b}$ (where m and b are real numbers) will have graphs that are straight lines.

Examples

Graph the equation $y = -2x$

Since this equation has the form $y = mx$ (m is just any real number), we know that its graph will be a straight line. We will need to find a minimum of 2 solutions to this equation. It is a good idea to plot more than 2 points. If the points don't "line up", you've made an arithmetic or plotting mistake. Check your solutions. To find some solutions to this equation, start by making a table.

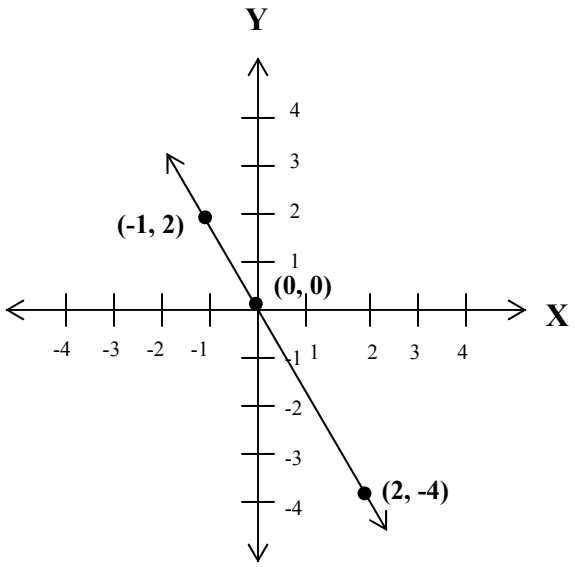
x	y = -2x	(x, y)

Pick any number you like for x. (Since you can pick any number, try to avoid fractions if you can or excessively large numbers that will take you off your paper.)

x	y = -2x	(x, y)
1	y = -2(1) = -2	(1, -2)
2	y = -2(2) = -4	(2, -4)
-1	y = -2(-1) = 2	(-1, 2)
0	y = -2(0) = 0	(0, 0)

Suppose we pick 1 for x. Put 1 in the x column. Now calculate y in the second column by replacing x with 1. Now fill in the third column with the ordered pair solution (1, -2). Continue this process until you have 2 points to plot (plus at least one more for good measure!). Plot these points on a coordinate system and draw a straight line through them.

Note: If your equation has the form $y = mx$, the graph will always go through the origin. If you pick 0 for x, $y = m(0) = 0$, so (0, 0) will always be on the graph.

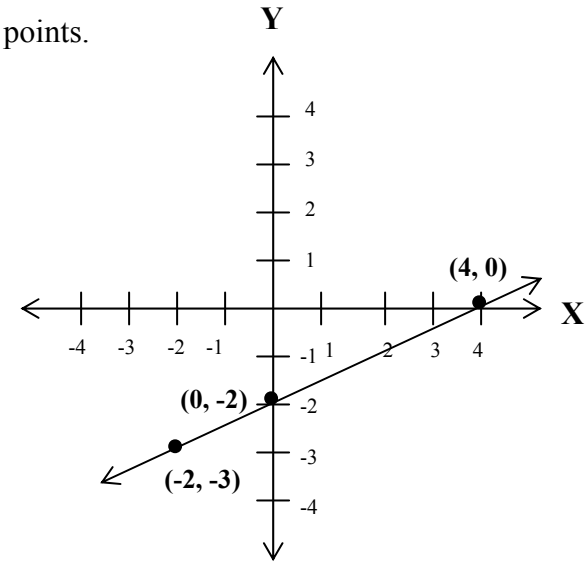


Graph the equation $y = \frac{1}{2}x - 2$

Since this equation has the form $y = mx + b$ (m and b are just real numbers), we know that its graph will be a straight line. Make a table. Since x is going to be multiplied by $\frac{1}{2}$, let's pick only even numbers for x (to avoid fractions).

x	$y = \frac{1}{2}x - 2$	(x, y)
-2	$y = \frac{1}{2}(-2) - 2$ $= -1 - 2$ $= -3$	(-2, -3)
0	$y = \frac{1}{2}(0) - 2$ $= 0 - 2$ $= -2$	(0 ,-2)
4	$y = \frac{1}{2}(4) - 2$ $= 2 - 2$ $= 0$	(4, 0)

Plot the points.

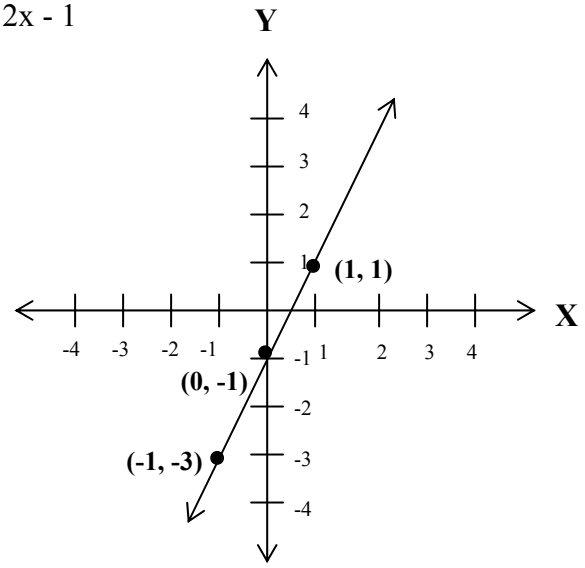


Graph the equation 2x - y = 1

Does this equation have the form $y = mx$ or $y = mx + b$? We will have to solve it for y to find out. Solving for y will also make it easier to find solutions.

$2x - y = 1$
$-2x \quad -2x$
$-y = -2x + 1$
$-1(-y = -2x + 1)$
$y = 2x - 1$

Yes. This equation has the form $y = mx + b$ so its graph will be a straight line



Make the table and plot the points.

x	y = 2x - 1	(x, y)
-1	y = 2(-1) -1 = -2 -1 = -3	(-1 ,-3)
0	y = 2(0) -1 = 0 -1 = -1	(0, -1)
1	y = 2(1) -1 = 2 -1 = 1	(1 , 1)

Problems

1. Plot these points: (0, 3), (-2, 4), (-1, -3), (3, 0)
2. In which quadrants are these points located?

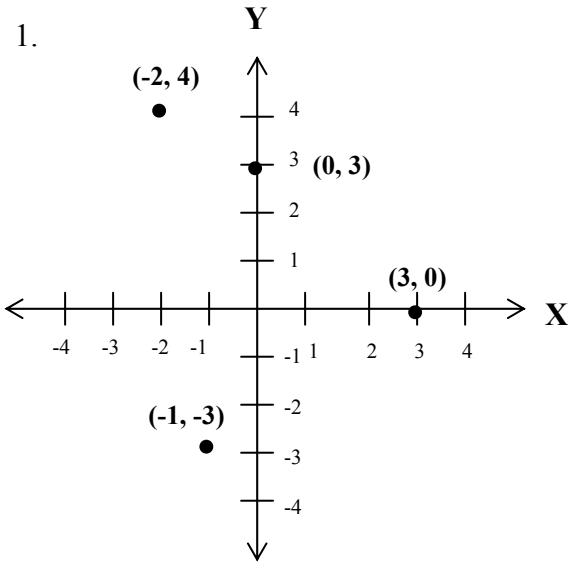
   a) (-2, -3) b) $(\frac{4}{3}, \frac{1}{2})$ c) (2.1, -4) d) (-1000, 421357)

3. Is the point (-1, -2) a solution of $2x - 3y = 3$?
4. Graph $y = \frac{5}{3}x$
5. Graph $y = -2x - 1$
6. Graph $5x + y = 7$
7. Graph $x - 2y = 4$

Answers

2. a) Quadrant III b) Quadrant I c) Quadrant IV d) Quadrant II 3. No

1.



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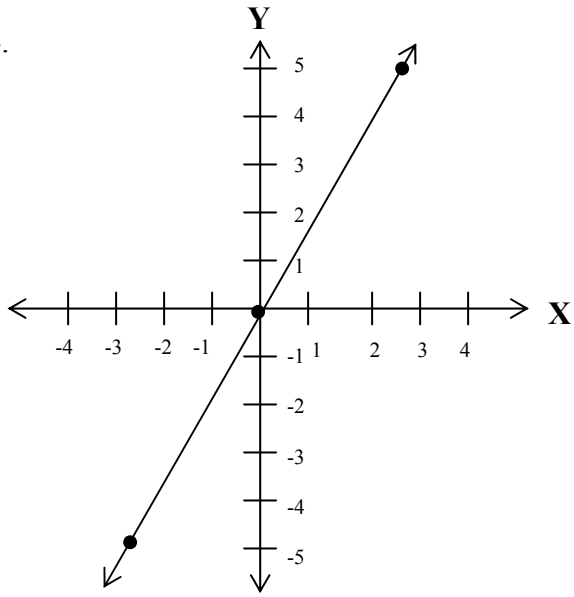

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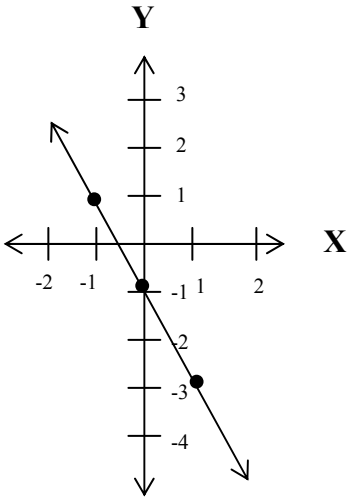

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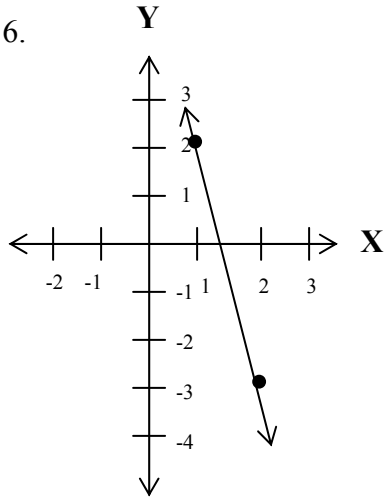

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