FRACTIONS (1.1)

Prime Number - a natural number that has exactly two different factors (the number itself and 1).
Prime Factorization - writing a number as a product of prime factors only.

Example: Find the prime factorization of 84

$84 = 4 \cdot 21$ Start with *any* factorization of 84.

$= 2 \cdot 2 \cdot 3 \cdot 7$ 4 is not a prime number, so factor it. 21 is not a prime number, so factor it. Continue this process until all factors are prime numbers. 2, 3, and 7 are prime numbers so the prime factorization of 84 is $2 \cdot 2 \cdot 3 \cdot 7$

Multiplying Fractions

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

Example $\quad \frac{2}{3} \cdot \frac{5}{7} = \frac{2 \cdot 5}{3 \cdot 7} = \frac{10}{21}$

Dividing Fractions

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

Example $\quad \frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \cdot \frac{7}{5} = \frac{2 \cdot 7}{3 \cdot 5} = \frac{14}{15}$

Simplifying Fractions

$$\frac{a \cdot b}{a \cdot c} = \frac{b}{c}$$

Example Multiply and simplify $\frac{4}{5} \cdot \frac{3}{6}$

$\frac{4}{5} \cdot \frac{3}{6} = \frac{4 \cdot 3}{5 \cdot 6}$ Use rule for multiplying fractions.

$= \frac{2 \cdot 2 \cdot 3}{5 \cdot 2 \cdot 3}$ Find the prime factorization for each number.

$= \frac{2 \cdot 2 \cdot \cancel{3}}{5 \cdot 2 \cdot \cancel{3}}$ Notice the factor of 3 in both numerator and denominator and the factor of 2 in both numerator and denominator. "Cancel" the matched pairs in numerator and denominator.

$= \frac{2}{5}$

Important Notes!!!

1. Any whole number can be written as a fraction by giving it a denominator of 1 (example $5 = \frac{5}{1}$)

2. The number 1 is a factor of every number. If <u>all</u> of your factors cancel out in the numerator or denominator, there is still a factor of 1 left.

Example: $\frac{2 \cdot 3}{5 \cdot 2 \cdot 3} = \frac{1 \cdot \cancel{2} \cdot \cancel{3}}{5 \cdot \cancel{2} \cdot \cancel{3}} = \frac{1}{5}$ NOT $\frac{0}{5}$

3. Only FACTORS of numerator and denominator can be cancelled!

$\frac{2+3}{3}$ IS NOT CORRECT! 2 PLUS 3 is NOT a factorization of the numerator.

FACTORS are MULTIPLIED by each other.

Adding and Subtracting Fractions

If denominators are same, simply add or subtract numerators, keep the common denominator in your answer

Example: $\frac{2}{3} + \frac{5}{3} = \frac{2+5}{3} = \frac{7}{3}$

If denominators are not same, **Example:** $\frac{4}{12} - \frac{1}{30}$

1. Do a prime factorization of each denominator. $\frac{4}{2\cdot 2\cdot 3} - \frac{1}{2\cdot 3\cdot 5}$

2. Use your prime factorization to determine a *least common denominator (LCD)*. The LCD must contain all of the factors of each denominator.

$$\text{LCD} = \overbrace{2 \cdot 2 \cdot 3}^{\text{factors of 12}} \cdot \underbrace{2 \cdot 3 \cdot 5}_{\text{factors of 30}}$$

3. Turn both denominators into the LCD by multiplying numerator and denominator by the appropriate (missing) factors.

$$\frac{4}{2\cdot 2\cdot 3}\cdot\frac{5}{5} - \frac{1}{2\cdot 3\cdot 5}\cdot\frac{2}{2}$$

4. Multiply. .

$$\frac{4\cdot 5}{2\cdot 2\cdot 3\cdot 5} - \frac{1\cdot 2}{2\cdot 3\cdot 5\cdot 2}$$

5. Do the multiplication in the numerators only.

$$\frac{20}{2\cdot 2\cdot 3\cdot 5} - \frac{2}{2\cdot 3\cdot 5\cdot 2}$$

6. Follow rules for adding and subtracting with like denominators.

$$= \frac{18}{2\cdot 2\cdot 3\cdot 5}$$

7. Factor and simplify if possible.

$$\frac{2\cdot 3\cdot 3}{2\cdot 2\cdot 3\cdot 5} = \frac{3}{10}$$

Examples

Write the prime factorization for 420.

$420 = 42 \cdot 10$
$= 6 \cdot 7 \cdot 2 \cdot 5$
$= 2 \cdot 3 \cdot 7 \cdot 2 \cdot 5$
$= 2 \cdot 2 \cdot 3 \cdot 5 \cdot 7$ (it's "nice" to arrange them in order)

Multiply or divide and simplify.

$$\frac{3}{2} \cdot \frac{20}{3} = \frac{3 \cdot 20}{2 \cdot 3} = \frac{3 \cdot 2 \cdot 2 \cdot 5}{2 \cdot 3} = \frac{\cancel{3} \cdot \cancel{2} \cdot 2 \cdot 5}{1 \cdot \cancel{2} \cdot \cancel{3}} = \frac{10}{1} = 10$$

$$\frac{3}{20} \div 3 = \frac{3}{20} \div \frac{3}{1} = \frac{3}{20} \cdot \frac{1}{3} = \frac{3 \cdot 1}{20 \cdot 3} = \frac{3 \cdot 1}{2 \cdot 2 \cdot 5 \cdot 3} = \frac{\cancel{3} \cdot 1}{2 \cdot 2 \cdot 5 \cdot \cancel{3}} = \frac{1}{20}$$

Add or subtract and simplify.

$$\frac{1}{8} + \frac{3}{8} = \frac{1+3}{8} = \frac{4}{8} = \frac{2 \cdot 2}{2 \cdot 2 \cdot 2} = \frac{\cancel{2} \cdot \cancel{2} \cdot 1}{\cancel{2} \cdot \cancel{2} \cdot 2} = \frac{1}{2}$$

$$\frac{5}{6} - \frac{3}{8} = \frac{5}{2 \cdot 3} - \frac{3}{2 \cdot 2 \cdot 2} \qquad \text{LCD} = 2 \cdot 2 \cdot 2 \cdot 3$$

$$= \frac{5}{2 \cdot 3} \cdot \frac{2 \cdot 2}{2 \cdot 2} - \frac{3}{2 \cdot 2 \cdot 2} \cdot \frac{3}{3}$$

$$= \frac{5 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 3} - \frac{3 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 3}$$

$$= \frac{20}{2 \cdot 2 \cdot 2 \cdot 3} - \frac{9}{2 \cdot 2 \cdot 2 \cdot 3}$$

$$= \frac{11}{2 \cdot 2 \cdot 2 \cdot 3}$$ No factors alike in numerator and denominator

$$= \frac{11}{24}$$

Problems

Find the prime factorization for:

1. 90
2. 147
3. 72

Perform the indicated operation and simplify if possible.

4. $\frac{1}{7} + \frac{3}{7}$
5. $\frac{7}{9} - \frac{4}{9}$
6. $\frac{1}{6} \cdot 8$
7. $\frac{2}{3} \div \frac{4}{3}$
8. $\frac{1}{4} + \frac{3}{2}$
9. $\frac{7}{6} - \frac{1}{4}$
10. $\frac{20}{45} + \frac{4}{18}$

Answers

1. $2 \cdot 3 \cdot 3 \cdot 5$
2. $3 \cdot 7 \cdot 7$
3. $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$
4. $\frac{4}{7}$
5. $\frac{1}{3}$
6. $\frac{4}{3}$
7. $\frac{1}{2}$
8. $\frac{7}{4}$
9. $\frac{11}{12}$
10. $\frac{2}{3}$