

## 2 Exponents, Order of Operations, and Inequality (1.2)

### Exponential Notation

$b^n$  means  $\underbrace{b \cdot b \cdot b \cdot b \cdot \dots \cdot b}_{n \text{ factors}}$

$$b^1 = b$$

$$b^0 = 1$$

### Examples

$$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$$

$$(-1)^5 = (-1) (-1) (-1) (-1) (-1) = -1$$

$$(1/3)^2 = (1/3) (1/3) = 1/9$$

$$2^1 = 2$$

$$(-4)^1 = -4$$

$$(95xyz)^0 = 1$$

### BE CAREFUL!!!

$(-2)^4$  IS NOT THE SAME AS  $-2^4$

$$(-2)^4 = (-2) (-2) (-2) (-2) = 16$$

$$-2^4 = -[(2) (2) (2) (2)] = -16$$

### Order of Operations

**1. Perform all calculations within grouping symbols.** If there are lots of parentheses, brackets, etc. - start with innermost grouping symbols and work to the outside. If it is not possible to do the calculations inside the grouping symbol, eliminate the parentheses by using the distributive law when appropriate.

**2. Evaluate all exponential expressions.**

**3. Perform all multiplications and divisions in order from left to right.**

**4. Perform all additions and subtractions in order from left to right.**

<b>Please</b>	<b>Excuse</b>	<b>My</b>	<b>Dear</b>	<b>Aunt</b>	<b>Sally</b>
(Parentheses)	(Exponents)	(Multiplication and	(Addition and Subtraction		
		Division Left to Right)	Left to Right)		

### SPECIAL NOTE:

The expression  $-(5 - x)$  is equivalent to the expression  $-1 \cdot (5 - x)$  so the distributive law applies to this situation.

$$-(5 - x) = -1 \cdot (5 - x) = (-1) \cdot (5) + (-1) \cdot (-x) = -5 + x$$

SHORTCUT: When “distributing” a minus sign, drop the parentheses and change the sign on every term inside the parentheses.

### Examples

1)  $(7 - 3 \cdot 4) - 2$

$$(7 - 12) - 2$$

$$(-5) - 2$$

$$-7$$

Do calculations within grouping symbols first. Inside the parentheses is a subtraction and a multiplication.

Do multiplication first,

then the subtraction.

Now do outside subtraction.

$$2) 3 \{ [-2(x^2 + 1) - 4] - [5(x^2 - 2) + 3] \}$$

$$3 \{ [-2x^2 - 2 - 4] - [5x^2 - 10 + 3] \}$$

$$3 \{ [-2x^2 - 6] - [5x^2 - 7] \}$$

$$3 \{ -2x^2 - 6 - 5x^2 + 7 \}$$

$$3 \{ -7x^2 + 1 \}$$

$$-21x^2 + 3$$

Start with innermost grouping symbol (parentheses). Since we cannot do the calculations inside the parentheses (not like terms) let's try to remove the parentheses by using the distributive law.

Do what calculations can be done within the square brackets.

Eliminate square brackets by using distributive law. First bracket does not have a minus sign in front so DO NOT change signs inside! Only second bracket has a minus in front.

Do calculations within curly brackets.

Eliminate curly brackets by using distributive law. Expression cannot be simplified further.

## Symbols of Inequality

The symbol  $\neq$  means “is not equal to”, as in  $5 \neq 7$  (“5 is not equal to 7”).

If two numbers are not equal, one must be larger than the other. The symbol  $<$  means “is less than”, as in  $5 < 7$  (“5 is less than 7”).

The symbol  $>$  means “is greater than”, as in  $7 > 5$  (“7 is greater than 5”).

Note that the pointy end of the “greater than” or “less than” symbol always points to the smaller number. A ‘less than’ statement can be changed to a ‘greater than’ statement by switching the values on either side of the inequality and reversing the symbol:  $9 < 12$  (“9 is less than 12”) can also be written as  $12 > 9$  (“12 is greater than 9”).

The symbol  $\leq$  means “is less than *or* equal to”, as in  $-3 \leq -1$  (“-3 is less than or equal to -1”). This statement is true because -3 is less than -1.

The symbol  $\geq$  means “is greater than or equal to”.  $5 \geq 5$  (“5 is greater than or equal to 5”) is a true statement, since  $5 = 5$ .

**$\leq$  and  $\geq$  statements are true if *either* the  $<$  or  $>$  part is true *or* the  $=$  part is true.**

## Problems

Simplify:

1.  $3^3$

5.  $4 + 2 \cdot 3 - 2^3 \cdot (-2)$

9.  $(x + 2) - (2x - 1)$

2.  $(-3)^3$

6.  $(4 - 2)^3 - 4 \cdot 3(-1)^2$

10.  $-3(2x - 2) + 4(-1 + x)$

3.  $-3^3$   
 $6)^2]$

7.  $-(4 - x)$

11.  $-4[2(x - 3) + 7 - 2(4 \cdot 2 -$

4.  $(2x)^3$

8.  $-2(-x + 3)$

12.  $-2[-3(2 + 1)^2 + 25]^2$

## Answers

1. 27

5. 26

9.  $-x + 3$

2. -27

6. -4

10.  $-2x + 2$

3. -27

7.  $-4 + x$

11.  $-8x + 28$

4.  $8x^3$

8.  $2x - 6$

12. -8