22 Complex Fractions (6.5)

If a rational expression has additional rational expressions in its numerator or denominator, it is called a *complex rational expression*.

For example: $\dfrac{\frac{1}{5}+\frac{2}{3}}{10}$, $\dfrac{\frac{a+b}{c}}{\frac{c}{a-b}}$, and $\dfrac{\frac{5}{x}-y}{\frac{x^2-y^2}{x+y}}$ are all complex rational expressions.

Your book describes two methods of simplifying these expressions. Method 2 involves simplifying the numerator and denominator separately, using common denominators unique to each part of the fraction. This worksheet describes Method 1 – using a denominator *common to all* the simple fractions in the complex numerator and denominator. This method is often faster because it clears groups of fractions earlier.

Example 1.

Let's try it with numbers only first. Add and simplify: $\dfrac{\frac{1}{3}+\frac{5}{6}}{\frac{1}{4}+\frac{3}{8}}$

Step 1. Find the LCD. The denominators of all the simple fractions are: 3, 6, 4, and 8. Find a denominator common to all four.
(Refer to Worksheet 6.2 – 6.5)

$$3 \qquad 6 \qquad 4 \qquad 8$$

$$3 \qquad 3 \bullet 2 \qquad 2 \bullet 2 \qquad 2 \bullet 2 \bullet 2$$

$$\searrow \quad \downarrow \quad \swarrow \quad \swarrow$$

$$3 \bullet 2 \bullet 2 \bullet 2$$

= 24 LCD

Step 2. Multiply top and bottom of the complex fraction by the LCD. $\dfrac{\frac{1}{3}+\frac{5}{6}}{\frac{1}{4}+\frac{3}{8}} \bullet \dfrac{24}{24}$

Which means, multiply every simple fraction in the complex fraction by the LCD.

$$\dfrac{\left(\frac{1}{3}\bullet\frac{24}{1}\right)+\left(\frac{5}{6}\bullet\frac{24}{1}\right)}{\left(\frac{1}{4}\bullet\frac{24}{1}\right)+\left(\frac{3}{8}\bullet\frac{24}{1}\right)}$$

Step 3. Divide the LCD by all the denominators: This eliminates the denominators in all the simple fractions.

$$\frac{\left(\frac{1}{\cancel{3}} \cdot \frac{\cancel{24}\,^{8}}{1}\right) + \left(\frac{5}{\cancel{6}} \cdot \frac{\cancel{24}\,^{4}}{1}\right)}{\left(\frac{1}{\cancel{4}} \cdot \frac{\cancel{24}\,^{6}}{1}\right) + \left(\frac{3}{\cancel{8}} \cdot \frac{\cancel{24}\,^{3}}{1}\right)}$$

Step 4. Multiply and simplify.

$$= \frac{(1 \bullet 8) + (5 \bullet 4)}{(1 \bullet 6) + (3 \bullet 3)}$$

$$= \frac{8+20}{6+9} = \frac{28}{15}$$

<u>Example 2.</u> Now with variables: perform the indicated operations and simplify.

$$\frac{1+\frac{2}{x}}{3-\frac{1}{x^3}}$$

Step 1. Find the LCD.
The denominators of all the simple fractions are: x and x^3.
The LCD is x^3 (the highest power of the variable).

$$\text{LCD} = x^3$$

Step 2. Multiply top and bottom by the LCD.

$$\frac{1+\frac{2}{x}}{3-\frac{1}{x^3}} \bullet \frac{x^3}{x^3}$$

$$= \frac{\left(1 \bullet x^3\right) + \left(\frac{2}{x} \bullet \frac{x^3}{1}\right)}{\left(3 \bullet x^3\right) - \left(\frac{1}{x^3} \bullet \frac{x^3}{1}\right)}$$

Step 3. Simplify the individual fractions and add/subtract.

$$= \frac{x^3 + \left(\frac{2}{\cancel{x}} \bullet \frac{x^{\cancel{3}\,2}}{1}\right)}{3x^3 - \left(\frac{1}{\cancel{x^3}} \bullet \frac{\cancel{x^3}}{1}\right)}$$

$$= \frac{x^3 + 2x^2}{3x^3 - 1}$$

Step 4. Can it be further simplified?

No. Done!

Example 3. Perform the indicated operations and simplify.

$$\frac{\frac{x+1}{x-5}+\frac{2}{x+3}}{\frac{x}{x+3}+\frac{x}{x-5}}$$

Step 1. Find the LCD for all the simple fractions. The denominators are (x – 5) and (x + 3), so the LCD is just those factors multiplied together:

LCD = (x – 5)(x + 3)

Step 2. Multiply top and bottom by the LCD.

$$\frac{\frac{x+1}{x-5}+\frac{2}{x+3}}{\frac{x}{x+3}+\frac{x}{x-5}} \bullet \frac{(x-5)(x+3)}{(x-5)(x+3}$$

$$= \frac{\left[\frac{x+1}{x-5}\bullet\frac{(x-5)(x+3)}{1}\right]+\left[\frac{2}{x+3}\bullet\frac{(x-5)(x+3)}{1}\right]}{\left[\frac{x}{x+3}\bullet\frac{(x-5)(x+3)}{1}\right]+\left[\frac{x}{x-5}\bullet\frac{(x-5)(x+3)}{1}\right]}$$

Step 3. Simplify the individual fractions.

$$= \frac{\left[\frac{(x+1)}{\cancel{(x-5)}}\bullet\frac{\cancel{(x-5)}(x+3)}{1}\right]+\left[\frac{2}{\cancel{(x+3)}}\bullet\frac{(x-5)\cancel{(x+3)}}{1}\right]}{\left[\frac{x}{\cancel{(x+3)}}\bullet\frac{(x-5)\cancel{(x+3)}}{1}\right]+\left[\frac{x}{\cancel{(x-5)}}\bullet\frac{\cancel{(x-5)}(x+3)}{1}\right]}$$

$$= \frac{(x+1)\,(x+3)\;+\;2\,(x-5)}{x\,(x-5)\;+\;x\,(x+3)}$$

Step 4. Multiply and add/subtract.

$$= \frac{x^2+4x+3+2x-10}{x^2-5x+x^2+3x}$$

$$= \frac{x^2+6x-7}{2x^2-2x}$$

Step 5. Can it be further simplified? Try factoring.

$$= \frac{(x+7)\,\cancel{(x-1)}}{2x\,\cancel{(x-1)}}$$

$$= \frac{x+7}{2x}$$

Problems

1. $$\frac{5-\frac{1}{4}}{6+\frac{2}{3}}$$

2. $$\frac{\frac{2}{x}+x}{\frac{x}{5}+x}$$

3. $$\frac{\frac{a^2-b^2}{ab}}{\frac{a+b}{b}}$$

4. $$\frac{\frac{2}{x-y}-\frac{2}{x}}{y}$$

5. $$\frac{\frac{x^2-x-12}{x^2-2x-3}}{\frac{x^2+9x+18}{x^2-4x+3}}$$

6. $$\frac{\frac{6}{x^2y}+\frac{3}{xy^4}}{\frac{3}{x^3y^2}-\frac{15}{xy}}$$

7. $$\frac{a^2-\frac{16}{a^2}}{a+\frac{8}{a^2}}$$ (Mat 102; optional for Mat 101)

8. $$\left[\frac{\frac{m+2}{m-2}+1}{\frac{m+2}{m-2}-1}\right]^4$$

Answers

1. $$\frac{57}{80}$$

2. $$\frac{5x^2+10}{6x^2}$$

3. $$\frac{a-b}{a}$$

4. $$\frac{2}{x^2-xy}$$

5. $$\frac{(x-4)(x-1)}{(x+6)(x+1)}$$

6. $$\frac{2xy^3+x^2}{y^2-5x^2y^3}$$

7. $$\frac{(a^2+4)(a-2)}{a^2-2a+4}$$

8. $$\frac{m^4}{16}$$