13 **Adding and Subtracting Polynomials (4.4)**

Examples of polynomials

0 $\sqrt{12}$ $\frac{x}{2}$ $-4x$ $7xy^2 + 2x^3 - 8$ $z^3 + 1$

Examples that are <u>not</u> polynomials (and why)

$\frac{3}{x^2} + 4x$ (variable in <u>denominator</u> of fraction) $\sqrt{x} - 1$ (<u>variable</u> under radical)

Special names

A polynomial with only one term is called a **monomial**. $-4xy^2$ is an example of a monomial.
A polynomial with only 2 terms is called a **binomial**. $5x + 2$ is an example of a binomial.
A polynomial with only 3 terms is called a **trinomial**. $\frac{7}{2}x^2 + 2y - xyz$ is an example of a trinomial.
A polynomial with more than 3 terms has no special name.

Degree

The degree of a **term** will be the same as the exponent on the <u>variable</u> (or the <u>sum</u> of the exponents if there is more than 1 variable in the term). The degree of the **polynomial** will be the same as its highest degree term. Consider this trinomial: $-2x^4 + \frac{3}{4}x^2y - 7$

The degree of the first term is 4 (the exponent on the variable is 4). The degree of the second term is 3 (there are 2 variables, x has an exponent of 2, y has an exponent of 1, the sum of the exponents is 3). The third term has a degree of 0. (There is no variable in this term. But $-7 = -7(1)$ and $x^0 = 1$ so you can think of the term as $-7x^0$.) The degree of this polynomial is 4 (the term with the largest degree is the first term with a degree of 4, so the polynomial is said to be of degree 4).

Like terms

Terms whose variable parts are the same are called **like terms** and can be "combined" (added or subtracted) by using the distributive law.

<u>Examples</u>

$5xy + 3xy$ These are like terms. The variable parts are the same (both xy).
$= (5 + 3)xy$ They can be added.
$= 8xy$

$2x^3y^2 - 7x^3y^2$ These are like terms. The variable parts are the same (both x^3y^2).
$= (2 - 7)x^3y^2$ They can be subtracted.
$= -5x^3y^2$

$\frac{4}{3}x^2y - \frac{1}{6}yx^2$ These are like terms. The variable parts are the same ($x^2y = yx^2$ by the commutative law of multiplication).

$= \frac{8}{6}x^2y - \frac{1}{6}yx^2$ They can be subtracted by finding a common denominator.

$= \frac{7}{6}x^2y$ OR $\frac{7}{6}yx^2$

$2xy - 3xz$ These are <u>not</u> like terms ($xy \neq xz$). They cannot be subtracted.

$5x^2y + 3xy^2$ These are <u>not</u> like terms ($x^2y \neq xy^2$). They cannot be added.

Simplify the following polynomial by combining like terms.

$$5x^3 - 4xy^2 + 3x^2y - xy^2 + 2x^3z - 2x^3$$

Identify like terms. It may help to write them next to each other.

$\underline{5x^3 - 2x^3}$ $\quad$ $\underline{-4xy^2 - xy^2}$ $+ 3x^2y + 2x^3z$

like terms $\quad$ like terms

Combine like terms. $3x^3 \quad - 5xy^2 \quad + 3x^2y + 2x^3z$

Adding and Subtracting Polynomials

To **add** polynomials, combine like terms.

<u>Example</u>:

$(2x^3 - 4x + 3zx) + (-2x + 6x^3 - xz + 8)$ The parentheses can be eliminated. They are serving no purpose other than to separate the 2 polynomials.

$= 2x^3 - 4x + 3zx + -2x + 6x^3 - xz + 8$

$= 2x^3 + 6x^3 - 4x - 2x + 3zx - xz + 8$ Group like terms together.

$= 8x^3 \quad -6x \quad + 2xz + 8$ Combine like terms.

It is considered proper form when writing a polynomial to arrange the terms so that the term with the highest degree is first, the term with the second highest degree is second, etc. This is called writing the polynomial in **descending order**. The above answer should be written as $8x^3 + 2xz - 6x + 8$.

To **subtract** polynomials, change to addition by changing the signs on the terms of the polynomial being subtracted.

Example

$(x^3 + 4y^2 - 7x + 4) - (5y^2 - z + 1 - x)$

↓ change signs of all terms being subtracted

change to addition

$= x^3 + 4y^2 - 7x + 4 + -5y^2 + z - 1 + x$

Treat as an addition problem by combining like terms.

$= x^3 + 4y^2 - 5y^2 - 7x + x + 4 - 1 + z$

$= x^3 - y^2 - 6x + 3 + z$

Arrange in descending order.

$= x^3 - y^2 - 6x + z + 3$ (or $x^3 - y^2 + z - 6x + 3$)

Example

Subtract $-z^2 y + 7z - y$ from $5 + 2y - 8z^2 y + 7z$

Write as $(5 + 2y - 8z^2 y + 7z) - (-z^2 y + 7z - y)$

$= 5 + 2y - 8z^2 y + 7z + z^2 y - 7z + y$

$= 5 + 2y + y - 8z^2 y + z^2 y + 7z - 7z$

$= 5 + 3y - 7z^2 y + 0$

$= -7z^2 y + 3y + 5$

Evaluating Polynomials

Consider this polynomial: $x^4 - 2x^3 - x^2 + 4x - 2$

You may be asked to **evaluate** this polynomial for a particular value (or values) of the variable (or variables). To evaluate this polynomial for $x = 2$, replace each x with 2. To avoid mistakes with exponents, place the 2 in parentheses first before you simplify.

$x^4 - 2x^3 - x^2 + 4x - 2$

$(2)^4 - 2(2)^3 - (2)^2 + 4(2) - 2$

$= 16 - 2(8) - 4 + 8 - 2$ (Remember! Do exponents before multiplication.)

$= 16 - 16 - 4 + 8 - 2$

$= 2$

Evaluate the polynomial for $x = -1$

$x^4 - 2x^3 - x^2 + 4x - 2$

$(-1)^4 - 2(-1)^3 - (-1)^2 + 4(-1) - 2$ (This shows why the parentheses are helpful. Do exponents before multiplication.)

$= 1 - 2(-1) - 1 - 4 - 2$

$= 1 + 2 - 1 - 4 - 2$

$= -4$

PROBLEMS

For each expression below, state whether or not it is a polynomial. If it <u>is</u> a polynomial, answer the following questions:

a) How many terms does it have?
b) Does it have a special name? If so, what?
c) State the degree of each term and the degree of the polynomial.

1) $8x - x^2 + 2 - 4x^3$
2) $-\frac{1}{2}$
3) $3x^2 - \frac{4}{x^3}$
4) $x^2 y^5 + z + 3$

Add (or subtract) the following polynomials. Write your answer in descending order.

5) $(2x^2 - x + 4) + (6x^2 - 4x + 2)$
6) $(x^3 - 0.3x^2) + (2x - 0.1x^2)$
7) $(2xy^2 - 7xy + x^2 y) + (10yx - x^2 y)$
8) $(3x^2 - 4x) - (2x^2 + x - 2)$
9) $(-z - 5x^3 z + 4) - (z - 3 + x^3 z)$

10) Evaluate $-2x^3 - x^2 + 3x - 1$ for $x = -2$
11) Evaluate $x^2 z^3 - x^3 z^2$ for $x = -1$ and $z = -3$

Answers

1) Polynomial
   a) 4 terms
   b) no special name
   c) terms have degree 1, 2, 0, and 3 respectively; degree of polynomial is 3
2) Polynomial
   a) 1 term
   b) monomial
   c) degree 0; degree of polynomial is 0
3) Not a polynomial
4) Polynomial
   a) 3 terms
   b) trinomial
   c) terms have degree 7, 1, and 0 respectively; degree of polynomial is 7

5) $8x^2 - 5x + 6$
6) $x^3 - 0.4x^2 + 2x$
7) $2xy^2 + 3xy$
8) $x^2 - 5x + 2$
9) $-6x^3 z - 2z + 7$
10) 5
11) -18