

Basic Matrix Manipulation with a Graphing Calculator

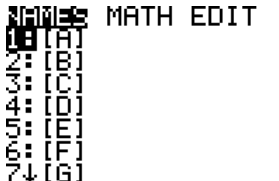
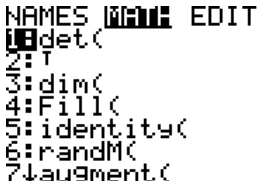
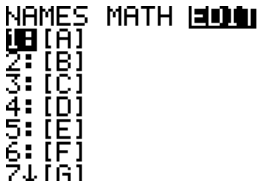
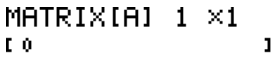
Often, a matrix may be too large or too complex to manipulate by hand. For these types of matrices, we can employ the help of graphing calculators to solve them.

We will cover a few of the most common graphing calculators used in education today. Throughout the directions, words appearing in **bold** indicate calculator buttons. To learn more, or if your calculator is not demonstrated, consult the manufacturer's product manual.

TI-83 Plus/84 Plus:

I will be using the TI-83 Plus graphing calculator for these directions. The TI-84 Plus family of graphing calculators is the upgraded version of the TI-83 Plus, with possible extra features in the menus demonstrated below. However, the older TI-83 and TI-82 graphing calculators have slightly different keyboard layouts than the TI-83 Plus, so these directions will not be entirely accurate for those calculators. For example, instead of pressing $2^{nd} \rightarrow \mathbf{MATRIX}$ to access the matrix menu, just press the dedicated **MATRIX** button.

Inputting/Editing Matrices:

<p>Before we can work with matrices, we must first input them into the calculator. To enter the Matrix Editor, press $2^{nd} \rightarrow \mathbf{MATRIX}$ (above the x^{-1} key). The TI-83/84 calculators contain predefined matrix variables labeled [A] through [J].</p>	
<p>The "NAMES" column in the Matrix Editor is where we will choose a matrix after it has been created. The "EDIT" column looks just like the "NAMES" column, except this is where we input or edit a matrix. The "MATH" column contains special commands that only work with matrices, some of which we will discuss later.</p>	
<p>Inputting matrices into the TI-83/84 is easy. Press the right arrow key twice over to the "EDIT" column. Use the UP and DOWN arrow keys to select a matrix name. Let's use [A] since it is already selected, so press ENTER.</p>	
<p>Since we are creating a new matrix, the calculator displays a default 1x1 matrix. Let's say we want a 3x2 matrix instead. Type: $3 \rightarrow \mathbf{ENTER} \rightarrow 2 \rightarrow \mathbf{ENTER}$.</p>	

You should now have this screen. As you can see, we have a 3x2 matrix filled with zeros. To input your own matrix, type in each element of the matrix

row-by-row. That is, to enter the matrix $\begin{bmatrix} 4 & 2 \\ 1 & 3 \\ 8 & 7 \end{bmatrix}$, type: **4** → **ENTER** → **2** →

ENTER → **1** → **ENTER** → **3** → **ENTER** → **8** → **ENTER** → **7** → **ENTER**.

```
MATRIX[A] 3 ×2
[0 0 ]
[0 0 ]
[0 0 ]

1,1=0
```

And this is what you should have. Before we can use this matrix, we need to first exit the Matrix Editor. To do this, do not press **CLEAR**! Pressing **CLEAR** will remove the selected number. Instead, press **2nd** → **QUIT** (above **MODE**) to return to the Home screen.

```
MATRIX[A] 3 ×2
[4 2 ]
[1 3 ]
[8 7 ]

3,2=7
```

Return to the Matrix Editor (**2nd** → **MATRIX**). Notice that [A] now has the dimension of our matrix beside it. This is how we know that a matrix has already been entered into the calculator. To edit this matrix, right-arrow-key over to "EDIT", select the matrix name ([A] in this case) and press **ENTER**. The stored matrix will be displayed for editing. Remember to always press **2nd** → **QUIT** to exit the Editor!

```
NAMES MATH EDIT
1:[A] 3×2
2:[B]
3:[C]
4:[D]
5:[E]
6:[F]
7↓[G]
```

Adding and Subtracting Matrices:

Enter two matrices into the calculator as shown above, one in [A] and one in

[B]. For this example, $[A] = \begin{bmatrix} 4 & 2 \\ 1 & 3 \\ 8 & 7 \end{bmatrix}$ and $[B] = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$. In order to use a

matrix in a computation, select the name in the "NAMES" column and press **ENTER**. This will paste the name onto the Home screen where we can use it.

```
NAMES MATH EDIT
1:[A] 3×2
2:[B] 3×2
3:[C]
4:[D]
5:[E]
6:[F]
7↓[G]
```

To add these two matrices together, do the following:

"NAMES" → select [A] → **ENTER** → **+** → "NAMES" → select [B] → **ENTER** → **ENTER**

If done correctly, you should see this screen.

```
[A]+[B]
[[5 4 ]
 [4 7 ]
 [13 13]]
```

Subtracting matrices is similar, except type a subtraction sign instead of an addition sign.

(Remember, matrices must be the same dimension in order to add or subtract them. The calculator will return an error if the dimensions are not the same.)

```
[A]-[B]
[[3 0 ]
 [-2 -1]
 [3 1 ]]
```

Multiplying Matrices:

<p>Matrix multiplication is easy on the TI-83/84. For scalar multiplication, multiply the number times the matrix just like multiplying two numbers together. For example, to multiply 3 times the matrix [A], type: 3 → multiply key → “NAMES” → select [A] → ENTER → ENTER.</p>	<pre>3*[A] [[12 6] [3 9] [24 21]]</pre>
<p>Multiplying two matrices together is just as easy. However, remember to have the correct matrix dimensions, otherwise the calculator will give a “Dimension Mismatch” error. For this example, $[A] = \begin{bmatrix} 4 & 2 \\ 1 & 3 \\ 8 & 7 \end{bmatrix}$ and $[B] = \begin{bmatrix} 1 & 2 & 5 \\ 3 & 4 & 5 \end{bmatrix}$. Try duplicating my screen by multiplying [A] and [B] together.</p>	<pre>[A]*[B] [[10 16 20] [10 14 5] [29 44 40]]</pre>
<p>Notice that sometimes with scalar multiplication, if the scalar is a fraction, we may get decimals in our answer. But what if we wanted fractions instead? At the top of the general MATH menu (press the MATH button under ALPHA), you will find the “Frac” command. Press ENTER to paste it to the Home screen. The calculator will place “Ans” before the command, indicating that the most recent answer will be converted to a fraction. Press ENTER to run the command. Or, we can compress this to one step by adding the “Frac” command at the end before pressing ENTER on the multiplication.</p> <p>Note that fractional scalars must be multiplied to matrices, not divided. That is, $(1/2)[B]$ is correct, but $[B]/2$ will return a “Data Type” error. This is correct since there is no matrix division.</p>	<pre>(1/2)[B] [[.5 1 2.5] [1.5 2 2.5]] Ans▶Frac [[1/2 1 5/2] [3/2 2 5/2]] (1/2)[B]▶Frac [[1/2 1 5/2] [3/2 2 5/2]] ERR:DATA TYPE 1:Quit 2:Goto</pre>

Calculating the Inverse:

<p>To calculate a matrix inverse, first input $\begin{bmatrix} 5 & 1 \\ 3 & 8 \end{bmatrix}$ as matrix [C] into the TI-83/84. (Of course the matrix must be square, otherwise the calculator will return an “Invalid Dimension” error). Don’t forget to 2nd → QUIT out of the Matrix Editor.</p>	<pre>MATRIX[C] 2 x2 [[5 1] [3 8]] z,z=8</pre>
<p>Select matrix [C] from the “NAMES” column and press ENTER to paste it onto the Home screen. Press the x^{-1} key (under the MATH button) for the inverse command. Press ENTER. Since the answer contains decimal numbers, choosing “Frac” from the MATH menu and pressing ENTER will convert the decimals to fractions.</p>	<pre>[C]⁻¹ [[.2162162162 ... [-.0810810811 ...] Ans▶Frac [[8/37 -1/37] [-3/37 5/37]]</pre>

Determinants:

Enter this matrix as matrix [A] in the calculator:

$$\begin{bmatrix} 4 & 2 & 1 \\ 5 & 7 & 2 \\ 1 & -3 & 5 \end{bmatrix}$$

To calculate the determinant of [A], enter the matrix menu and **RIGHT** arrow key over to "MATH". Select the first command, "det(", and press **ENTER** to paste it to the Home screen. Select matrix [A] from "NAMES", press **ENTER** to paste it to the screen, and **ENTER** again to run the command. That's it! The determinant of this matrix is 96.

```
NAMES [MATH] EDIT
1:det(
2:†
3:dim(
4:Fill(
5:identity(
6:randM(
7:augment(

det([A]          96
■
```

Transpose:

Let's calculate the transpose of $\begin{bmatrix} 4 & 2 & 1 \\ 5 & 7 & 2 \\ 1 & -3 & 5 \end{bmatrix}$. If it is not there already,

input this matrix as [A] in the calculator. After pasting [A] to the Home screen from the "NAMES" column, go back to the "MATH" column in the matrix menu and select the second command. Press **ENTER** to paste the command to the screen, and then **ENTER** again to run the command.

```
NAMES [MATH] EDIT
1:det(
2:†
3:dim(
4:Fill(
5:identity(
6:randM(
7:augment(

[A]†
      [[4 5 1 ]
       [2 7 -3]
       [1 2 5 ]]
■
```

Deleting Matrices from the Calculator:

Deleting matrices from the TI-83/84 is not too hard, but we must be careful. We will be entering an area of the calculator where we could possibly delete something that we did not intend to. First, press **2nd → MEM** (above the "+" key). Choose the second item in the list, which stands for "Memory Management and Delete". Press **ENTER**. Arrow-key down to "Matrix..." and press **ENTER**. You should now see a list of all of the matrices stored on your calculator. Use the **UP** and **DOWN** arrow keys to move the little triangle to the matrix you want to delete, and then press the **DEL** key. Make sure to press **2nd → QUIT** to return to the Home screen.


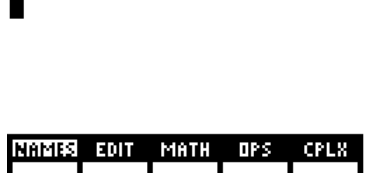
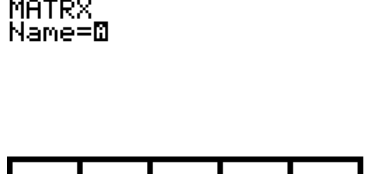

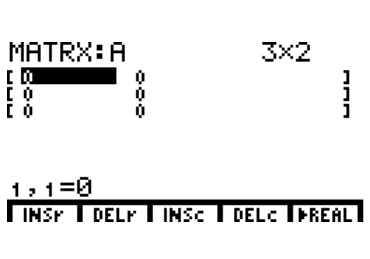

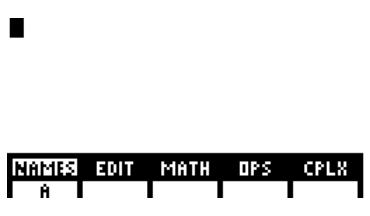
```
MEM [MEM]
1:About
2:Mem Mgmt/Del...
3:Clear Entries
4:ClrAllLists
5:Archive
6:UnArchive
7:Reset...

RAM FREE   24036
ARC FREE   163840
1:All...
2:Real...
3:Complex...
4:List...
5:Matrix...
6:Y-Vars...

RAM FREE   24036
ARC FREE   163840
▶ [A]      92
  [B]      65
  [C]      47
```

TI-86:

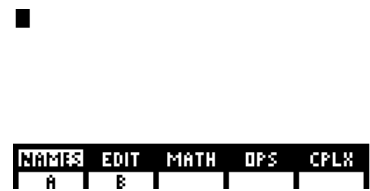
Inputting/Editing Matrices:

<p>Before we can work with matrices, we must first input them into the calculator. To enter the Matrix Editor, press 2nd → MATRIX (above the 7 key). Unlike the TI-83/84 calculators, the TI-86 does not contain any predefined matrix variables. Instead, it allows you to name the matrices yourself, limited to eight characters.</p>	
<p>The “NAMES” submenu in the Matrix Editor is where we will choose a matrix after it has been created. The “EDIT” submenu is similar to the “NAMES” submenu, except this is where we input or edit a matrix. The “MATH” and “OPS” submenus contain special commands that work with matrices, some of which we will discuss later.</p>	
<p>Inputting matrices into the TI-86 is relatively easy. Press the F2 key to enter the “EDIT” submenu. We first need to enter a name for the matrix. For simplicity, let’s name this matrix “A”. Since the calculator is already in ALPHA lock, pressing the LOG key will type “A”. Press ENTER.</p>	
<p>Since we are creating a new matrix, the calculator displays a default 1x1 matrix. Let’s say we want a 3x2 matrix instead. Type: 3 → ENTER → 2 → ENTER.</p>	
<p>You should now have this screen. As you can see, we have a 3x2 matrix filled with zeros. To input your own matrix, type in each element of the matrix row-by-row. That is, to enter the matrix $\begin{bmatrix} 4 & 2 \\ 1 & 3 \\ 8 & 7 \end{bmatrix}$, type: 4 → ENTER → 2 → ENTER → 1 → ENTER → 3 → ENTER → 8 → ENTER → 7 → ENTER.</p>	
<p>And this is what you should have. Before we can use this matrix, we need to first exit the Matrix Editor. To do this, do <u>not</u> press CLEAR! Pressing CLEAR will remove the selected number. Instead, press 2nd → QUIT (above EXIT) to return to the Home screen.</p>	
<p>Return to the Matrix Editor (2nd → MATRIX) and press F1. Notice that matrix “A” is now listed in the “NAMES” submenu. To edit this matrix, press EXIT to leave the “NAMES” submenu, then press F2 to enter the “EDIT” submenu. Press the key underneath “A” (in this case, F1) and then ENTER. The stored matrix will be displayed for editing. Remember to always press 2nd → QUIT to exit the Editor!</p>	

Adding and Subtracting Matrices:

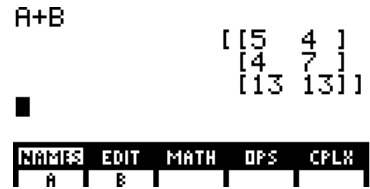
Enter two matrices into the calculator as shown above, one as “A” and one as “B”. For this example, $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \\ 8 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$. In order to

use a matrix in a computation, press the key underneath the matrix name in the “NAMES” submenu. This will paste the name onto the Home screen where we can use it.



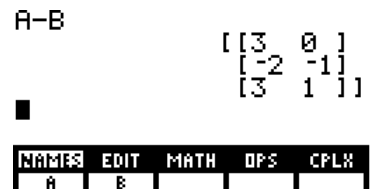
To add these two matrices together, do the following:
 “NAMES” submenu → F1 → + → F2 → ENTER

If done correctly, you should see this screen.



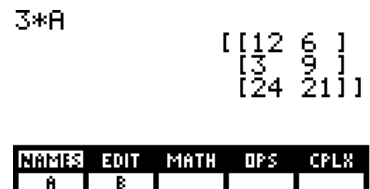
Subtracting matrices is similar, except type a subtraction sign instead of an addition sign.

(Remember, matrices must be the same dimension in order to add or subtract them. The calculator will return an error if the dimensions are not the same.)



Multiplying Matrices:

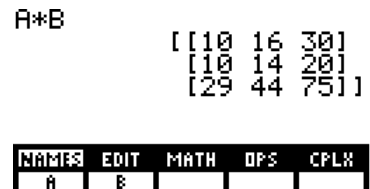
Matrix multiplication is easy on the TI-86. For scalar multiplication, multiply the number times the matrix just like multiplying two numbers together. For example, to multiply 3 times the matrix “A”, type:
3 → multiply key → “NAMES” submenu → F1 → ENTER.



Multiplying two matrices together is just as easy. However, remember to have the correct matrix dimensions, otherwise the calculator will

give a “Dimension Mismatch” error. For this example, $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \\ 8 & 7 \end{bmatrix}$ and

$B = \begin{bmatrix} 1 & 2 & 5 \\ 3 & 4 & 5 \end{bmatrix}$. Try duplicating my screen by multiplying “A” and “B” together.



Press **2nd** → **QUIT** to exit the Editor. Let's first try solving the system using the Gaussian Elimination method. Enter into the Matrix menu and press **F4** to enter the "OPS" submenu. Press **F4** again to paste the "ref" command ("Row-Echelon Form") onto the Home screen.

```

NAMES EDIT MATH OPS CPLX
dim Fill ident ref rref

```

Press **EXIT** then **F1** to switch to the "NAMES" submenu. Press **F1** to paste matrix "A" into the "ref" command. Before pressing **ENTER** to perform the computation, add the "Frac" command at the end to display possible fractions in the answer. If a "..." appears in the answer, this means that the matrix extends beyond the calculator's screen; use the **LEFT** and **RIGHT** arrow keys to scroll the matrix.

```

ref A>Frac
[[1 2 1/4 1/2 ]
 [0 1 -3/14 -2/7]
 [0 0 1 6 1]]

```

Note that if you swap rows, the matrix given by the "ref" command may not match the matrix obtained by performing the Gaussian elimination by hand. That is fine; the final solution will be the same.

```

NUM PROB ANGLE HYP MISC
rFrac 2 rEval *I eval

```

Now let's try the Gauss-Jordan elimination method. If you have calculated this method by hand, then you know that the answer will be obtained at the end. The same applies to the TI-86.

Return to the "OPS" submenu in the Matrix menu, but this time press **F5** to paste the "rref" command ("Reduced Row-Echelon Form") onto the Home screen.

```

NAMES EDIT MATH OPS CPLX
dim Fill ident ref rref

```

Select matrix "A" as before, with "Frac" for good measure, and press **ENTER**. There we go! The solution to our system of equations is (-3, 1, 6).

```

rref A>Frac
[[1 0 0 -3]
 [0 1 0 1 ]
 [0 0 1 6 ]]

```

Now what if we have a dependent or inconsistent system? The "rref" command will still work on the augmented matrices of these systems, but with different results than above. For a dependent system, a matrix with the last row all zeros would be returned (first picture on the right). For an inconsistent system, a matrix would be returned where the last row contains all zeros except for a final element of "1" (second picture on the right).

```

rref A>Frac
[[1 0 1 -2]
 [0 1 1 -3]
 [0 0 0 0 ]]

```

```

rref A>Frac
[[1 0 3 0]
 [0 1 5 0]
 [0 0 0 1]]

```

Determinants:

Input this matrix as matrix "A" in the calculator:

$$\begin{bmatrix} 4 & 2 & 1 \\ 5 & 7 & 2 \\ 1 & -3 & 5 \end{bmatrix}$$

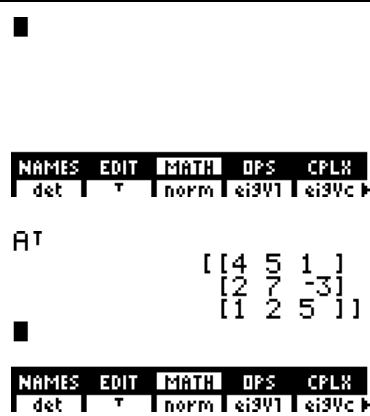
To calculate the determinant of "A", enter the Matrix menu and press **F3** for the "MATH" submenu. Press **F1** to paste the "det" command onto the Home screen. Next, press **EXIT** then **F1** to enter the "NAMES" submenu. Finally, press **F1** to paste matrix "A" to the screen, and then **ENTER** to run the command. That's it! The determinant of this matrix is 96.



Transpose:

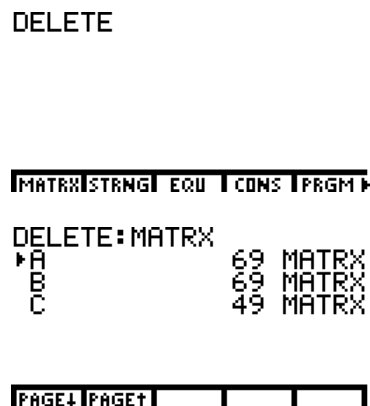
Let's calculate the transpose of $\begin{bmatrix} 4 & 2 & 1 \\ 5 & 7 & 2 \\ 1 & -3 & 5 \end{bmatrix}$. If it is not there

already, input this matrix as "A" in the calculator. After pasting matrix "A" to the Home screen from the "NAMES" submenu, go back to the "MATH" submenu and press **F2** for the "Transpose" command. Press **ENTER** to compute the answer.



Deleting Matrices from the Calculator:

Deleting matrices from the TI-86 is not too hard, but we must be careful. We will be entering an area of the calculator where we could possibly delete something that we did not intend to. First, press **2nd** → **MEM** (above the "3" key). Press **F2** for "Delete", press **MORE** once, and then **F1** for "Matrix". You should now see a list of all of the matrices stored on your calculator. Use the **UP** and **DOWN** arrow keys to move the little triangle to the matrix you want to delete, and then press the **ENTER** key. Make sure to press **2nd** → **QUIT** to return to the Home screen.

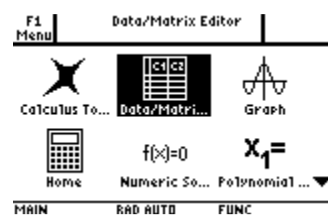


TI-89/TI-92/Voyage 200:

I will be using the TI-89 graphing calculator for these directions. The TI-92 and Voyage 200 are virtually the same, yet the menus may be slightly different due to their wider displays. I will be utilizing the Apps Desktop available in the latest firmware for these three calculators. If it is not turned on by default, press **MODE** and scroll all the way down to the last setting. Turn on the Apps Desktop and press **ENTER** to save the change.

Inputting/Editing Matrices:

Before we can work with matrices, we must first input them into the calculator. To enter the Matrix Editor, scroll over to the “Data/Matrix Editor” icon on the Apps Desktop and press **ENTER**.



Since we want to enter a new matrix, scroll down to “New...” and press **ENTER**.



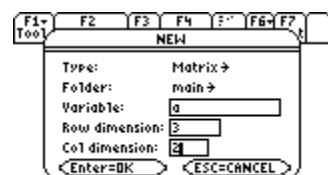
TYPE OR USE ←+1+ [ENTER] OR [ESC]

The Data/Matrix Editor allows us to input into the TI-89 more than just matrices. We can also enter lists and data (essentially a matrix without fixed dimensions).



USE ← AND → TO OPEN CHOICES

Let’s say we want a 3x2 matrix. We first need to tell the Editor that we want to input a matrix. At the “Type” option, press the **RIGHT** arrow key, select “Matrix” and press **ENTER**. Press the **DOWN** arrow key twice to “Variable” and press **□** to type an “a”. Press the **DOWN** arrow key to “Row dimension”. Since, the calculator is still in alpha lock from the “Variable” box, press **alpha** once to exit that lock. Now press **3** → **→** → **2** to enter the matrix dimensions. Press **ENTER** once to confirm the column dimension, then again to create the matrix.

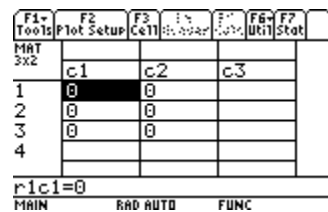


MAIN RAD AUTO FUNC

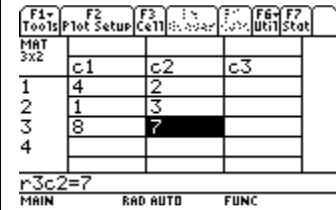
You should now have this screen. As you can see, we have a 3x2 matrix filled with zeros. To input your own matrix, type in each element of the

matrix row-by-row. That is, to enter the matrix $\begin{bmatrix} 4 & 2 \\ 1 & 3 \\ 8 & 7 \end{bmatrix}$, type: **4** →

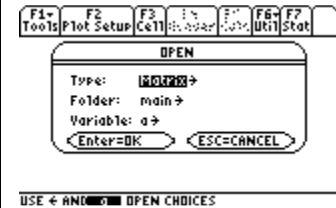
ENTER → **2** → **ENTER** → **1** → **ENTER** → **3** → **ENTER** → **8** → **ENTER** → **7** → **ENTER**.



And this is what you should have. Before we can use this matrix, we need to first exit the Data/Matrix Editor. To do this, do not press **[CLEAR]**! Pressing **[CLEAR]** will remove the selected number. Instead, press the **[HOME]** button to exit to the Home screen, or the **[APPS]** button to return to the Apps Desktop.



To edit the matrix, return to the Data/Matrix Editor, but this time select "Open". Change the type to "Matrix", make sure "a" is the selected variable, and press **[ENTER]**. The matrix "a" will now be displayed for editing. Remember to always press **[HOME]** or **[APPS]** to exit the Editor!

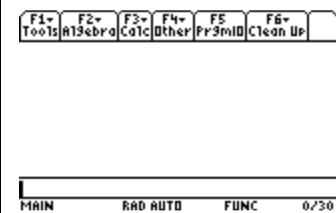


What if we need to change the matrix dimensions? After opening the matrix, press **[2nd]** → **[F1]** to display the "Utilities" menu and scroll down to "Resize Matrix"; press **[ENTER]**. Type in the new dimensions and press **[ENTER]** to apply them.



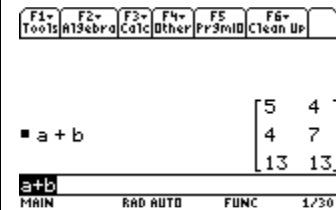
Adding and Subtracting Matrices:

Enter two matrices into the calculator as shown above, one as "a" and the other as "b". For this example, $a = \begin{bmatrix} 4 & 2 \\ 1 & 3 \\ 8 & 7 \end{bmatrix}$ and $b = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$. Make sure you are at the Home screen while using matrices in computations.



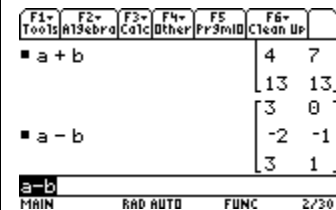
To add these two matrices together, do the following:
[alpha] → **[=]** (for "A") → **[+]** → **[alpha]** → **[(]** (for "B") → **[ENTER]**

If done correctly, you should see this screen.



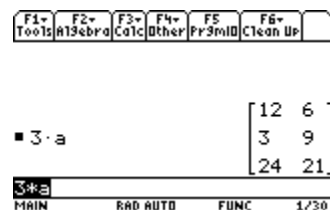
Subtracting matrices is similar, except type a subtraction sign instead of an addition sign.

(Remember, matrices must be the same dimension in order to add or subtract them. The calculator will return an error if the dimensions are not the same.)



Multiplying Matrices:

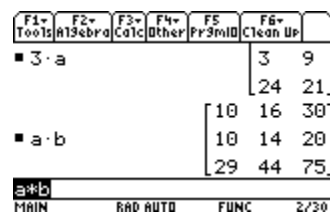
Matrix multiplication is easy on the TI-89. For scalar multiplication, multiply the number times the matrix just like multiplying two numbers together. For example, to multiply 3 times the matrix “a”, type: $3 \rightarrow \times \rightarrow \alpha \rightarrow “A” \rightarrow \text{ENTER}$.



Multiplying two matrices together is just as easy. However, remember to have the correct matrix dimensions, otherwise the calculator will give a

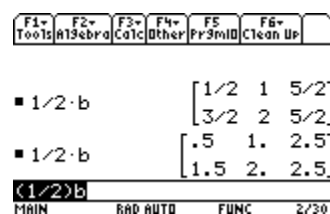
“Dimension” error. For this example, $a = \begin{bmatrix} 4 & 2 \\ 1 & 3 \\ 8 & 7 \end{bmatrix}$ and $b = \begin{bmatrix} 1 & 2 & 5 \\ 3 & 4 & 5 \end{bmatrix}$.

Try duplicating my screen by multiplying “a” and “b” together.



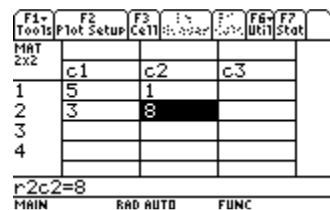
Notice that if the scalar is a fraction, the TI-89 will return fractions in the answer. If decimals are desired, press $\blacklozenge \rightarrow \text{ENTER}$ after the initial multiplication.

Unlike some other TI graphing calculators with matrices, the TI-89 understands “b/2” to be the same as “(1/2)b”.

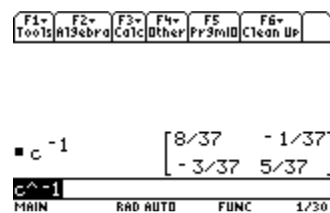


Calculating the Inverse:

To calculate a matrix inverse, first input $\begin{bmatrix} 5 & 1 \\ 3 & 8 \end{bmatrix}$ as matrix “c” into the TI-89. (Of course the matrix must be square, otherwise the calculator will return a “Dimension” error). Press HOME to exit to the Home screen.



Press $\alpha \rightarrow “C” \rightarrow \wedge \rightarrow (-) \rightarrow 1 \rightarrow \text{ENTER}$.

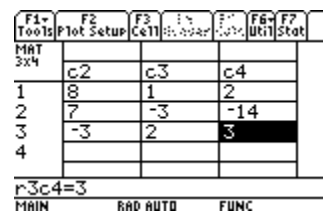


Gaussian and Gauss-Jordan Elimination:

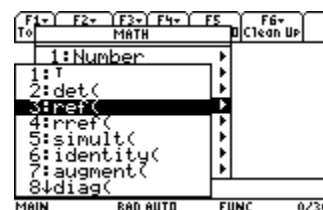
Let's use these two methods to solve the following system of equations:

$$\left. \begin{array}{l} 4x + 8y + z = 2 \\ x + 7y - 3z = -14 \\ 2x - 3y + 2z = 3 \end{array} \right\} \rightarrow \begin{bmatrix} 4 & 8 & 1 & 2 \\ 1 & 7 & -3 & -14 \\ 2 & -3 & 2 & 3 \end{bmatrix}$$

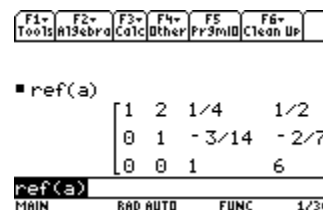
Input the augmented matrix as matrix "a".



Press **HOME** to exit the Editor. Let's first try solving the system using the Gaussian Elimination method. Press **2nd** → **5** to enter the calculator's Math menu. Scroll down to "Matrix", press the **RIGHT** arrow key, and select the "ref(" command ("Row-Echelon Form"). Press **ENTER** to paste the command to the Home screen.



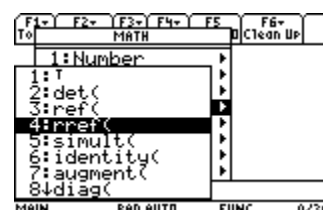
Type **alpha** → "A" → **]** → **ENTER** to run the command on matrix "a". If a "►" appears in the answer, this means that the matrix extends beyond the calculator's screen; select the answer and use the **LEFT** and **RIGHT** arrow keys to scroll the matrix.



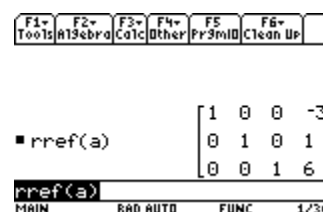
Note that if you swap rows, the matrix given by the "ref(" command may not match the matrix obtained by performing the Gaussian elimination by hand. That is fine; the final solution will be the same.

Now let's try the Gauss-Jordan elimination method. If you have calculated this method by hand, then you know that the answer will be obtained at the end. The same applies to the TI-89.

Press **2nd** → **5** to return to the Math menu, but this time scroll down to the "rref(" command ("Reduced Row-Echelon Form"), directly below "ref(". Press **ENTER** to paste the command to the Home screen.



Type **alpha** → "A" → **]** → **ENTER** to run the command on matrix "a". There we go! The solution to our system of equations is (-3, 1, 6).



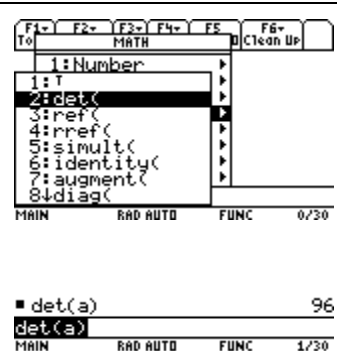
Now what if we have a dependent or inconsistent system? The “rref(“ command will still work on the augmented matrices of these systems, but with different results than above. For a dependent system, a matrix with the last row all zeros would be returned (first picture on the right). For an inconsistent system, a matrix would be returned where the last row contains all zeros except for a final element of “1” (second picture on the right).

$$\begin{aligned} \blacksquare \text{rref}(a) & \begin{bmatrix} 1 & 0 & 1 & -2 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ \blacksquare \text{rref}(a) & \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

Determinants:

Input this matrix as matrix “a” in the calculator: $\begin{bmatrix} 4 & 2 & 1 \\ 5 & 7 & 2 \\ 1 & -3 & 5 \end{bmatrix}$

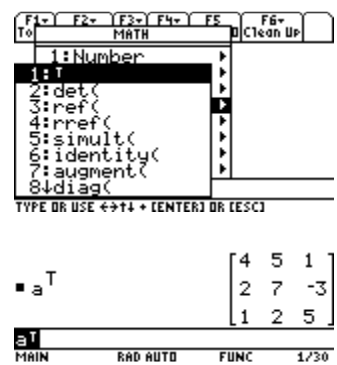
To calculate the determinant of “a”, press $\text{2nd} \rightarrow \text{5}$ to enter the calculator’s Math menu, scroll down to “Matrix”, press the **RIGHT** arrow key and select the “det(“ command. Press ENTER to paste the command to the Home screen. Type $\alpha \rightarrow \text{“A”} \rightarrow \text{)} \rightarrow \text{ENTER}$ to run the command on matrix “a”. That’s it! The determinant of this matrix is 96.



Transpose:

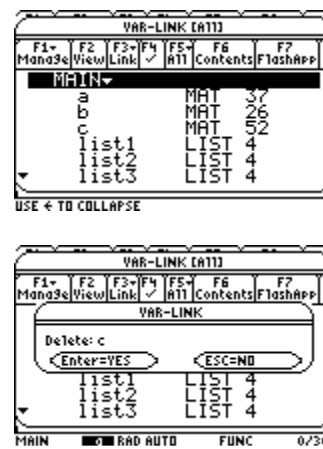
Let’s calculate the transpose of $\begin{bmatrix} 4 & 2 & 1 \\ 5 & 7 & 2 \\ 1 & -3 & 5 \end{bmatrix}$. If it is not there already,

input this matrix as “a” in the calculator. Press $\alpha \rightarrow \text{“A”}$ to type “a” on the Home screen, then go back to the Math menu, enter the “Matrix” submenu and select the first command. Press ENTER to paste the command to the screen, and then ENTER again to run the command.



Deleting Matrices from the Calculator:

Deleting matrices from the TI-89 is not too hard, but we must be careful. We will be entering an area of the calculator where we could possibly delete something that we did not intend to. First, press **2nd** → **VAR-LINK** (above the **□** key). Use the **UP** and **DOWN** arrow keys to select the matrix you want to delete, and then press the **←** key. Press **ENTER** to confirm or **ESC** to cancel. Make sure to press **HOME** to return to the Home screen.



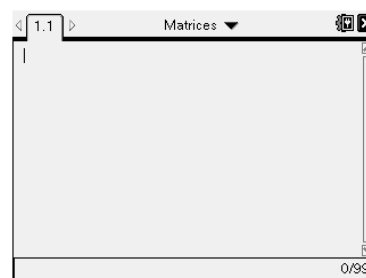
TI-Nspire:

I will be using a TI-Nspire CAS with Clickpad running software version 2.1 for these directions. The menus and buttons may differ slightly between the Clickpad, Touchpad, and different software versions.

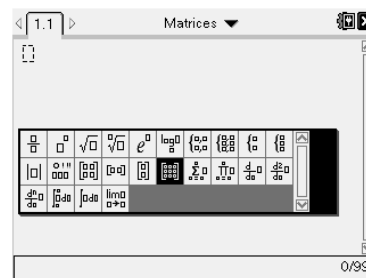
Inputting/Editing Matrices:

The TI-Nspire family of calculators provides a very easy way to input matrices directly onto the Calculator screen. This gives the advantage of not having to navigate through menus to enter and edit a matrix.

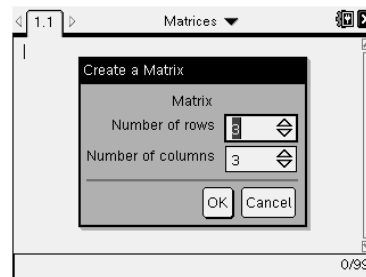
Begin by adding a calculator page to an existing document, or by creating a new document and choosing "Add Calculator".



Press **ctrl** → **⌘** to display the Template window. If necessary, move the selection to the 3x3 matrix symbol and press **enter**. This template will allow us to create a matrix of any size.

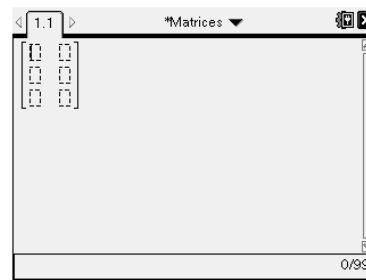


Since we are creating a new matrix, the calculator displays a default setting of three rows and three columns. Let's say we want a 3x2 matrix instead. Press tab to switch to the "Number of columns" box and type a "2". Press enter .

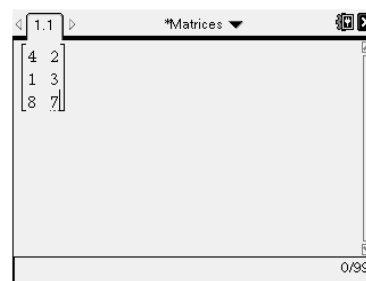


As you can see, we now have a 3x2 empty matrix waiting on the screen. Use a combination of numbers and arrow keys to fill the

template with the matrix $\begin{bmatrix} 4 & 2 \\ 1 & 3 \\ 8 & 7 \end{bmatrix}$.

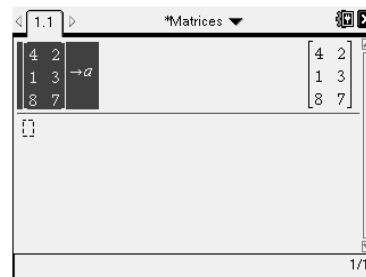
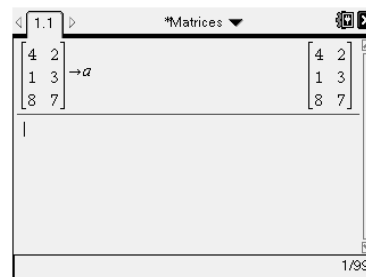


And this is what you should have. Now we can use this matrix as is, or we can store it for repeated use. To store it, move the cursor out all the way to the right side of the matrix, then press $\text{ctrl} \rightarrow \text{sto} \rightarrow \text{var}$. Type the letter "a" and press enter .



What we just did is to store the matrix into a variable titled "a". When we need to use this matrix, we can now just use the variable "a" instead of having to retype the matrix again.

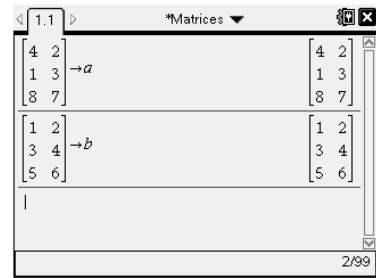
To edit this matrix, press the **UP** arrow key twice until the matrix with the arrow is selected, and then press enter to paste it to the next line. Edit the values in the matrix and press enter to store it as variable "a". If a whole new matrix with a different dimension is needed, then create the matrix as shown above and store it as variable "a".



Adding and Subtracting Matrices:

Enter two matrices into the calculator as shown above, one as “a” and

the other as “b”. For this example, $a = \begin{bmatrix} 4 & 2 \\ 1 & 3 \\ 8 & 7 \end{bmatrix}$ and $b = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$.

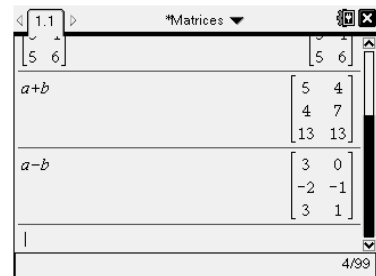


To add these two matrices together, do the following:

(A) → **(+)** → **(B)** → **(=)**

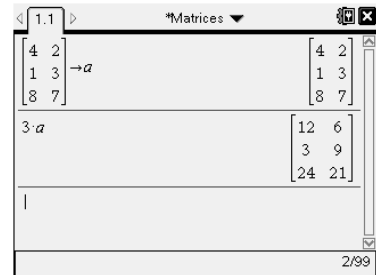
Subtracting matrices is similar, except type a subtraction sign instead of an addition sign.

(Remember, matrices must be the same dimension in order to add or subtract them. The calculator will return an error if the dimensions are not the same.)



Multiplying Matrices:

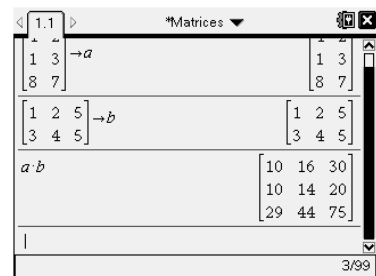
Matrix multiplication is easy on the TI-Nspire. For scalar multiplication, multiply the number times the matrix just like multiplying two numbers together. For example, to multiply 3 times the matrix “a”, type: **(3)** → **([a])** → **(A)** → **(=)**.



Multiplying two matrices together is just as easy. However, remember to have the correct matrix dimensions, otherwise the calculator will

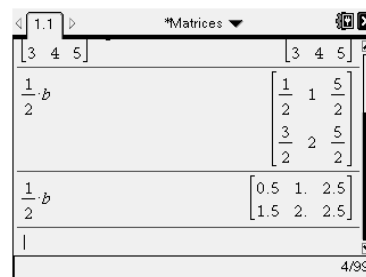
give a “Dimension” error. For this example, $a = \begin{bmatrix} 4 & 2 \\ 1 & 3 \\ 8 & 7 \end{bmatrix}$ and $b =$

$\begin{bmatrix} 1 & 2 & 5 \\ 3 & 4 & 5 \end{bmatrix}$. Try duplicating my screen by multiplying “a” and “b” together.



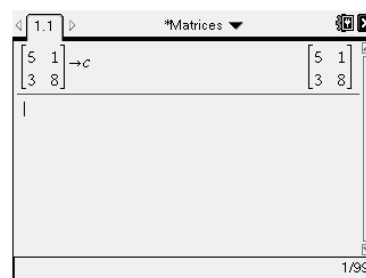
Notice that if the scalar is a fraction, the TI-Nspire will return fractions in the answer. If decimals are desired, press ctrl \rightarrow enter after the initial multiplication.

Unlike some other TI graphing calculators with matrices, the TI-Nspire understands “b/2” to be the same as “(1/2)b”.

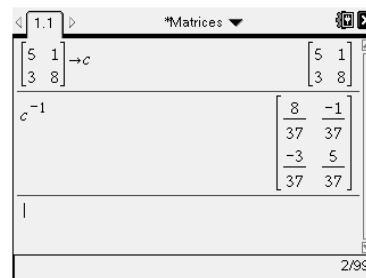


Calculating the Inverse:

To calculate a matrix inverse, first input $\begin{bmatrix} 5 & 1 \\ 3 & 8 \end{bmatrix}$ as matrix “c” into the TI-Nspire. (Of course the matrix must be square, otherwise the calculator will return a “Dimension” error).



Press C \rightarrow sqrt \rightarrow ans \rightarrow 1 \rightarrow enter .

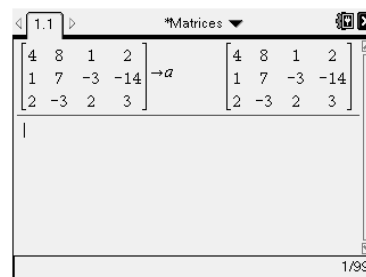


Gaussian and Gauss-Jordan Elimination:

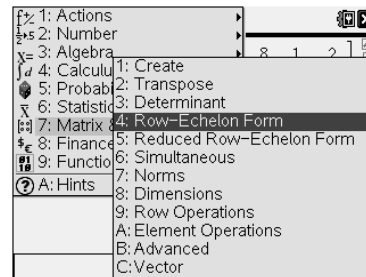
Let’s use these two methods to solve the following system of equations:

$$\left. \begin{array}{l} 4x + 8y + z = 2 \\ x + 7y - 3z = -14 \\ 2x - 3y + 2z = 3 \end{array} \right\} \rightarrow \begin{bmatrix} 4 & 8 & 1 & 2 \\ 1 & 7 & -3 & -14 \\ 2 & -3 & 2 & 3 \end{bmatrix}$$

Store the augmented matrix as matrix “a”.

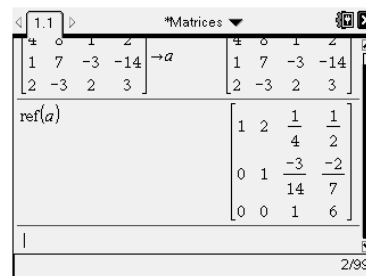


Let's first try solving the system using the Gaussian Elimination method. Press the **(menu)** button, scroll down to "Matrix & Vector", press **(enter)** (or **RIGHT** arrow key), and scroll down to "Row-Echelon Form". Press **(enter)**.



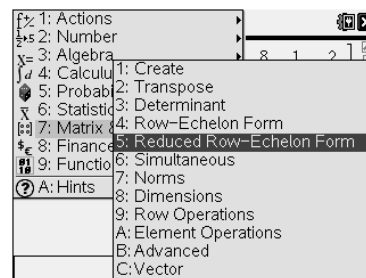
Press the **(A)** button and then **(enter)**.

Note that if you swap rows, the matrix given by the "ref(" command may not match the matrix obtained by performing the Gaussian elimination by hand. That is fine; the final solution will be the same.

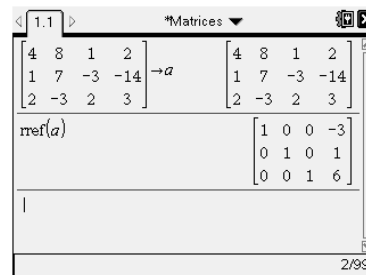


Now let's try the Gauss-Jordan elimination method. If you have calculated this method by hand, then you know that the answer will be obtained at the end. The same applies to the TI-Nspire.

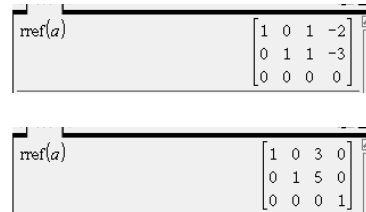
Press the **(menu)** button, scroll down to "Matrix & Vector", press **(enter)** (or **RIGHT** arrow key), and scroll down to "Reduced Row-Echelon Form". Press **(enter)**.



Press the **(A)** button and then **(enter)**. There we go! The solution to our system of equations is (-3, 1, 6).



Now what if we have a dependent or inconsistent system? The "rref(" command will still work on the augmented matrices of these systems, but with different results than above. For a dependent system, a matrix with the last row all zeros would be returned (first picture on the right). For an inconsistent system, a matrix would be returned where the last row contains all zeros except for a final element of "1" (second picture on the right).

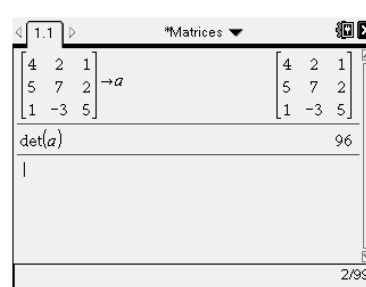


Determinants:

Store this matrix as matrix “a” in the calculator:

$$\begin{bmatrix} 4 & 2 & 1 \\ 5 & 7 & 2 \\ 1 & -3 & 5 \end{bmatrix}$$

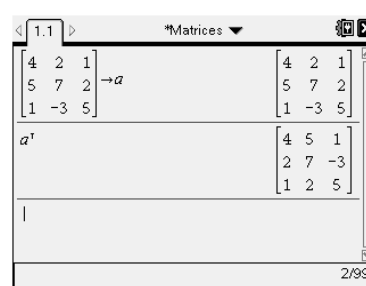
To calculate the determinant of “a”, press the **(menu)** button, scroll down to “Matrix & Vector”, press **(enter)** (or **RIGHT** arrow key), and scroll down to “Determinant”; press **(enter)**. Press the **(A)** button and then **(enter)**. That’s it! The determinant of this matrix is 96.



Transpose:

Let’s calculate the transpose of $\begin{bmatrix} 4 & 2 & 1 \\ 5 & 7 & 2 \\ 1 & -3 & 5 \end{bmatrix}$. If it is not there

already, input this matrix as “a” in the calculator. Press **(A)**, then **(menu)**, scroll down to “Matrix & Vector”, press **(enter)** (or **RIGHT** arrow key), and scroll down to “Transpose”; press **(enter)**. Press **(enter)** to run the command.



Deleting Matrices from the Calculator:

Deleting matrices is easy on the TI-Nspire. Press **(sto>var)** to display the list of variables stored in the document. Notice that the TI-Nspire identifies the matrix variables by displaying a little matrix icon beside the name. To delete one of these variables, press **(menu)**, select “Actions”, then press **(enter)** (or **RIGHT** arrow key), and scroll down to “Delete Variable”; press **(enter)**. Press **(sto>var)** to display the variable list and select the matrix you want to delete; press **(enter)** to paste the name to the screen. Press **(enter)** to run the command.

