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FROM THE CHAIR

Above all I think we should teach mathematics in a way that encourages students to think logically and analytically. Learning to think this way is one of the great values of studying mathematics. At all costs we should avoid encouraging students to follow any thought process that is sloppy, undiscerning and illogical. And yet modern mathematical notation is so cleverly designed as to sometimes tempt us to fall into the trap of doing just that. Indeed, I think there are times when we tend to insidiously lead students to conclusions that are perfectly true, neglecting to give the correct logical justification for the conclusion. Instead we offer some clever notational manipulation that "leads" to the correct conclusion, but that actually makes no sense at all.

Here is a case in point. Did you ever solve an antidifferentiation problem for a student by saying something like the following?

Find $\int 2x \sin x^2 \, dx$.

Let $y = x^2$, (1)
then $dy = 2xdx$. (2)

Therefore,

$$\int 2x \sin x^2 \, dx = \int \sin y \, dy \qquad (3)$$

$= -\cos y + C = -\cos x^2 + C$.

Of course the conclusion is correct, and the argument that "leads" to it seems eminently reasonable as well, just a matter of substituting equals for equals and using the integration formula for the integral of the sine function. There is one small problem. In spite of the ominous presence of the word "therefore," (3) does not logically follow from (1) and (2). It is not a substitution of equals for equals. The dy in (2) is not the same as the dy in (3). The dy in (2) is the differential of a dependent variable given by the definition:

> Let $y = f(x)$ and dx be a nonzero real valued number, then $dy = f'(x)dx$.

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(This comes up on Page 222 of our calculus book if you're interested.) But the dy in (3) is part of the notation used to denote the antiderivative of a function. Specifically, it denotes the independent variable in the function whose antiderivative is being represented. (This notation is introduced at the bottom of page 242 in our calculus text under *Notation for Antiderivatives.)* These two definitions for dy are entirely independent. It is a clever ploy of mathematicians to use the same symbol for these two different things, thereby intimating

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(Continued from page 1) that one can be substituted for the other as was done in the example above. But the fact that two different things are called by the same name hardly justifies replacing one for the other, and to let students get away with thinking this is to encourage sloppy, undiscerning and illogical thought.

Of course the true justification for the substitution in the example is that (3) is an application of the theorem that states that if g is differentiable and f is continuous and y = g(x), then

$$\int f(g(x))g'(x)dx = \int f(y)dy$$

The theorem is verified by differentiating both sides of the above equation with respect to x and employing the chain rule, not by substituting equals for equals that are not really equal!

Another example of this notational "sleight of hand" occurs in the solution of exact differential equations. At the bottom of page 37 in our differential equations text, the author introduces the exact differential equation

$$y\,dx + x\,dy = d(xy) = 0. \qquad (4)$$

He has noted that the expression y dx + x dy is equal to the total differential of the function of two variables z = xy. (This, together with the zero on the right side, is what makes the equation exact.) He then goes on to say, "By integrating we immediately obtain the implicit solution xy = c". First of all, it is not clear what he means for us to integrate. If y is a function of one variable, i.e., y = f(x), and we have dy = 0, it is true to conclude that y = c, because **if dy = 0, then y' dx = 0, and hence y' = 0, and hence y = c (by integrating).**

But the same reasoning cannot be applied to (4) because the expression xy is not a function of one but of two independent variables. If we tried to follow the same steps as above we would get that **if dz = 0, then z_x dx + z_y dy = 0** (using the definition of the differential of a function of two variables) **and hence ...** But there is no analogy to the next step.

So when he talks about integrating, I suppose what he has in mind is something like the following:

$$\text{Since } dy = 0 \text{ gives } \int dy = \int 0,$$
$$\text{which gives } y = c, \qquad (5)$$

then, in like manner,

$$dz = 0 \text{ gives } \int dz = \int 0,$$
$$\text{and then } z = c. \qquad (6)$$

Though the conclusion in (6) is correct, the logic offered for it, "By integrating", is suspect at best. Beside the fact that the expression $\int 0$ is meaningless because it does not denote the integration variable, we have the more troublesome fact that $\int dz$ has no meaning if z is a function of two variables. But it looks a lot like $\int dy$, which equals y when y is a function of one variable, so we are deceived into concluding that $\int dz$ equals z, even though z is a function of two variables, and $\int dz$ is devoid of meaning. This is another example of a clever but meaningless notational trick.

Al Giambrone ■

Conferences/Meetings

If you would like some new ideas to enhance your teaching or to learn something new about an area of mathematics or an opportunity to meet and exchange information with math colleagues abroad, then we recommend that you consider attending one of the many annual regional or national math conferences/meetings. These are excellent opportunities to expand your horizons mathematically or bring new or stimulating experiences into the classroom. We have shared here a few such events that are being held nearby this spring or summer.

1. Spring Section Meeting – Ohio Section of MAA
   Columbus, OH
   The Ohio State University
   April 4 & 5, 2003

Attend stimulating lectures on various mathematical topics. Invited addresses include Introductory Coding Theory, The Mathematics of Doodling, Mathematics at the National Security Agency, Math Connections, and Why the Golden Mean? A number of contributed papers (fifteen minute presentations) on topics of general mathematical interest or related areas will also be given. More information can be obtained by going to the Ohio Section website at www.maa.org/Ohio.

2. Spring Meeting of OhioMATYC
   Oregon, OH
   Maumee Bay State Park
   May 2 & 3, 2003

OhioMATYC meetings are great places to get feedback on your latest extra-effort project and may trigger a new line of thought for colleagues. Talks are from 15 to 60 minutes. More information is given on the General Info link on the OhioMATYC website at www.terra.edu/ohiomatyc.

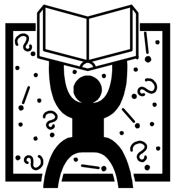
3. Michigan T3 Regional Institute
   Adrian, MI
   Siena Heights University
   May 9 & 10, 2003

The technology-based conference emphasis will be on enhancing the teaching and learning of mathematics using hand-held technology. Speakers will provide outstanding sessions that will give you ideas to take back to your classroom, and get you thinking about new ways to approach old problems. Whether you are a new or experienced user of technology, sessions are full of ideas to take back to the classroom and designed to help integrate technology into the classroom. Early registration is $40. More information can be obtained by going to the conference website at http://education.ti.com/us/t3/conferences/regional/adrian.html.

4. Summer Short Course – Ohio Section of MAA
   Columbus, OH
   Capital University
   July 16 – 18, 2003

In this course, designed for those with absolutely no experience in the subject, students will investigate techniques of encryption and decipherment along with the mathematical underpinnings of these systems. Discussed will be various symmetric encryption schemes (affine, keyword, the Vigenère Square, Playfair's System and Hill's System) as well as the RSA implementation of public key cryptography. Students will have ample opportunity to encrypt, decrypt and decipher messages as part of the course. Registration is $150. More information can be obtained by going to the Ohio Section website at www.maa.org/Ohio.

DEPARTMENT COLLOQUIUM



We will have a Department Colloquium on Friday, May 9, 2003 at 2:30 p.m. in Room 1001. All members of our full- and part-time faculty are welcome, as well as students who are interested in mathematics. The speaker and title are as follows:

Mr. Tom Wilson, Professor of Mathematics,
Sinclair Community College
"Surds, gradients, trapeziums, and other things: What I learned while teaching mathematics at a 6th form college in Manchester, England."

Refreshments will be served.

Harvey's Joke Corner

Math reduces stress:

$$\frac{2}{10} = \frac{1}{5}$$

Where to graduate in one's class:

Barber College – "head" of the class

Bartender's College – "fifth" in your class

Journalism School – 4th in your class (Fourth estate)

Krispy Kreme Baker's College – finish in the top dozen students

Sinclair Community College – 1st in your class (live next door to the school)

Harvey Chew ■

REMINDERS

- Don't forget that you will need to access your own class rosters electronically next quarter. You should have received a flier and letter from Al Giambrone with instructions on how to do this. For more information, contact the Math Office.

- Please do not permit students to sit in on your class who are not registered.

- Be sure to use incomplete grades in the appropriate way. Information is given on page 4.2 in our handbook. If you need a copy, contact the office.

- Please remember to turn in textbooks that you will not need next quarter. We are especially in need of math 101 books.