

# *Sinclair Mathnet*

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## FROM THE CHAIR



For some reason it seemed like a quiet uneventful year as we were going through it, but now as I look over all the things that have transpired these last months I am amazed at the many good things that have happened in the department this year.

One new full-time faculty member, Jim Hardman, and eleven new part-time faculty members, Lynn Allen, Phil Bantz, Jim Carruth, Alice Kemper, Earl King, Jason Lincoln, Charles Painter, Dahyabhai Patel, Greg Reese, Robert Shock and Jeff Willets joined our ranks this year. In addition, we had the pleasure of enjoying Peter Gill's service as a full-time member of our department on exchange from Ridge Danyers College in Manchester, England, while our own Tom Wilson spent a year teaching in Peter's place in the Mathematics Department at Ridge Danyers. And of course one of the interesting outcomes of this exchange was the fact that as one of our three departmental colloquia this year we were able to hold a live video conference with the Mathematics Department at Ridge Danyers in February.

And then there were all the awards and grants. Bob Chaney traveled to Florida to receive the Award for Innovative Excellence in Teaching, Learning and Technology, sponsored by the League for Innovation, for his work in incorporating activity based components into Statistics and Tech Math courses, development of "SAM the Robot" as a tool for delivering this new curriculum, and equipping room 10-327 and establishing the Math and Science Technology Center as facilities for teaching these courses. Then Bob, Barb Carruth and Kay Cornelius, along with Fred Thomas from physics, won the college's coveted Innovator of the Year

Award for their work in the development of activities and lab manuals for use in the activity based components of Statistics and Tech Math and for pioneering the teaching of these courses. (See page 3.) They will travel to California next year to be honored. (You can see these two awards on display in our display case outside the department office. By the way, the installation of the display case and the displays presented in it by David Stott are also among the many happy new events of this year.) Not choosing to rest on their laurels, Bob and Kay proceeded to win a National Science Foundation grant for \$400,000 over two years to offer workshops and disseminate information nationally on the creation and use of "SAM the Robot" for activity based mathematics education. Vickie Lair has also gotten in on the act by being named a co-principal investigator for another National Science Foundation grant for \$90,000 per year for four years to provide scholarships for students majoring in computer science, engineering and mathematics. And Susan Myers and her committee this year completed work on their \$50,000 Learning Challenge grant to explore alternative technology based approaches to delivering Math 101.

We continued to provide special services to our students. Again this year we conducted three rounds of mathematics competitions sponsored by AMATYC under the direction of David Stott and myself. Thirty students participated in at least one of these rounds. Susan Harris had excellent participation in her Problem of the Week Competition in Winter and Spring Quarters with 68 participants. Once again new attendance records were set in the Math Lab and Math Help Room. The total visit count for Fall and Winter Quarters combined rose to 12,414 visits – a 17.7% increase over last year. And Marie Stroh has continued to offer the Math Success and (Continued on Page 8)



## Faculty Feature



Marilyn Thompson receives the Part-time Faculty Member of the Year Award from Al Giambrone at the Spring Quarter Department Colloquium.

The recipient of this year's Part-time Faculty Member of the Year Award is Marilyn Thompson. Marilyn started teaching for Sinclair in 1986, and has taught a wide variety of courses including Math 101, 102, 116, 105 and 121.

Marilyn is presently an intervention specialist at Oakwood High School. She works with freshmen and sophomores who, for a variety of reasons, are not successful in the regular curriculum. She has been in this position since 1975.

Most of the Sinclair classes that Marilyn has taught have been at off-campus locations such as Centerville, Kettering, and Wright Patterson. These locations have offered unique challenges. Marilyn says, "Develop a sense of humor. You'll need it for tough times—like when there are scheduling goofs at Centerville High School, and you find hundreds of cheerleaders in the commons area outside your classroom which has no door." She goes on to add, "But you'll have fewer tough times if your students laugh WITH you and not AT you. I purposefully incorporate humor into my classroom presentation as well as spontaneously letting it happen."

Her favorite class to teach is Math 116, College Algebra. She likes to get students working together. "Students build relationships with other students, and then end up teaching one another in

this course and in future courses. I arrive to find pairs of students showing one another how to work the homework including pointing out one another's mistakes."

Marilyn has been in the Dayton area since 1968, and she received both Bachelor's and Master's degrees from Wright State University. She had taken some classes at Sinclair and enjoyed her experiences. When someone suggested she apply to teach here, she did so. Marilyn has had very positive teaching experiences at Sinclair, and says, "I have loved my Sinclair experience. I joked once that I would 'almost' do it for free, but I am glad with retirement coming that I did not do that!"

Marilyn's husband, Malcolm, is an art teacher in the Kettering schools. She and her husband travel and camp in the summer, when they go on hikes looking for wildlife such as bears, alligators, wild turkeys and cranes. She is the sponsor of the Oakwood high school's ecology club, so as she travels she seeks out environmental news and interesting places to visit. When she is home during warm months, she is an "eager" hosta gardener, dividing her plants, trading with other hosta collectors, and buying new varieties.

Congratulations on your well-deserved award, Marilyn!

Susan Harris ■

## REMINDERS

- Please remember to turn in books you will not be using next quarter so that others can use them.
- Be sure to use Incomplete grades in the appropriate way. Information is given on this on page 4.2 in the Mathematics Department Part-time Faculty Handbook. If you need a copy please contact the Office.
- Students complain frequently about teachers who "don't teach". They just "read from the textbook and do examples that are in the book." Please remember it is important to enrich, clarify and give structure to the material presented in the book by presenting the definitions, theorems, concepts, principles and procedures in organized lectures that use examples and illustrations in addition to the ones already given in the text.



**Innovator of the Year**

This spring Barbara Carruth, Bob Chaney, Kay Cornelius and Fred Thomas received Sinclair Community College's annually awarded Innovator of the Year Award. They received the award at the annual Employee Recognition Dinner Saturday, May 18, and on Thursday, May 23, they were given a special reception hosted by President Sifferlen.

The team won the award for their innovative use of activities and technology in the statistics and technical mathematics courses. With the acquisition and development of the Math-Science Technology Center in Rooms 11-441 and 11-442, and later the conversion of Room 10-327 to a lab/classroom, the statistics and tech math courses have incorporated hands-on, collaborative learning activities to help motivate course content and both clarify and deepen students' understanding of theoretical principles. The activities have provided students with a more diverse learning experience than the traditional classroom, and they have required students to become more actively engaged in the learning process.



Kay Cornelius, Barb Carruth, Bob Chaney and Fred Thomas

By learning and applying mathematics simultaneously, students have become more excited about mathematics, and they better understand where mathematics is and can be used.

The team will travel to Long Beach, California, in November to attend the 2002 Conference on Information Technology, which is sponsored by the League for Innovation in the Community College.

Barb, Bob, Fred and Kay, congratulations on a well-deserved honor!



## What's in a Definition?

In teaching calculus, I have learned that students do not always properly understand the definitions of the derivative and integral. The definitions involve the concept of a limit that they have only recently studied. After covering a number of examples in class using the definitions, when they learn about the basic differentiation rules or the integration rules and the Fundamental Theorem, they are inclined to want to depart from the definitions altogether and enjoy the rules, the rules that are intrinsically based on these precise definitions.

Well, we might not be able to get students to fully appreciate the definitions, but it would be nice to get them to at least understand them. For instance, in the case of the derivative, students can let their graphical / intuitive reasoning get in the way of knowing and applying the correct mathematical definition.

The definition students learn for the derivative is as follows:

The derivative of  $f$  at  $c$  is given by

$$f'(c) = \lim_{\Delta x \rightarrow 0} \frac{f(c + \Delta x) - f(c)}{\Delta x}$$

provided the limit exists.

Alternately, the derivative of  $f$  at  $c$  is given by

$$f'(c) = \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$$

provided the limit exists.

These definitions involve the idea of taking the limit of the slopes of secant lines that pass through the points  $(c, f(c))$  and  $(c + \Delta x, f(c + \Delta x))$  (or through  $(c, f(c))$  and  $(x, f(x))$  for the alternate definition). The tangent line to the graph of  $f$  at  $(c, f(c))$  is defined as the line whose slope is this limit and that passes through the point  $(c, f(c))$ .

Using this definition, it can be shown that if  $f$  is differentiable at  $c$  (i.e.,  $f'(c)$  is a real number), then  $f$  is continuous at  $c$ . It also implies that derivatives have the intermediate value property. (See *Mathnet*, Vol.8, Issue 2, page 5.) Students learn that there can be numbers  $c$  in the domain of a function at which the function may not be differentiable, such as for  $f(x) = |x|$  when  $c = 0$  since the one-sided derivatives are different, and for  $f(x) = \sqrt[3]{x}$  when  $c = 0$  because the left-hand and right-hand derivatives are both (positively) infinite. The latter example leads to a discussion of vertical tangent lines, which are tangent lines at  $(c, f(c))$  such that

$$\lim_{\Delta x \rightarrow 0} \frac{f(c + \Delta x) - f(c)}{\Delta x} = \infty$$

or

$$\lim_{\Delta x \rightarrow 0} \frac{f(c + \Delta x) - f(c)}{\Delta x} = -\infty . *$$

The definition for vertical tangent lines can also be extended to the endpoints of a closed interval by considering the appropriate one-sided limits. (\**Calculus, Sixth Ed.*, Larson, Hostetler and Larson, page 94.)

It was in showing a class that  $f(x) = \sqrt{x}$  has a vertical tangent line at  $(0,0)$  by looking at the right-hand derivative at 0, that I realized they did not properly understand the definition of the derivative.

The class saw that the derivative of  $f$  is  $f'(x) = \frac{1}{2\sqrt{x}}$  for  $x \neq 0$ , so it was clear that we could not use this formula to determine  $f'(0)$ . However, after I showed them that

$$\lim_{\Delta x \rightarrow 0^+} \frac{f(0 + \Delta x) - f(0)}{\Delta x} =$$

$$\lim_{\Delta x \rightarrow 0^+} \frac{\sqrt{0 + \Delta x} - \sqrt{0}}{\Delta x} =$$

$$\lim_{\Delta x \rightarrow 0^+} \frac{1}{\sqrt{\Delta x}} = \infty ,$$



one student pointed to the previously derived formula for the derivative of  $f$  for  $x \neq 0$  and asked whether we had shown there is a vertical tangent line since the limit of the derivatives as  $x$  approaches 0 in this formula is infinite, that is,

$$\lim_{x \rightarrow 0} f'(x) = \infty .$$

I answered by saying that we had rather shown that the limit of the slopes of the secant lines is infinite, but I did not dwell on it too long because I could tell she seemed convinced that by examining

$$f'(x) = \frac{1}{2\sqrt{x}},$$

and more particularly  $\lim_{x \rightarrow 0} f'(x)$ , one could see that the derivative is infinite at 0. Since both the limits were infinite, I just moved on.

This difference in understanding is very subtle, yet very profound. The question here is whether one can find a derivative at  $c$  by taking the limit of the surrounding derivatives as you approach  $c$ . For the functions our students study, this is usually true. This is because we work with elementary functions that usually have continuous derivatives. If a function  $f$  is differentiable at  $c$ , and if  $f'$  is continuous at  $c$ , then it is true that

$$f'(c) = \lim_{x \rightarrow c} f'(x).$$

However, if  $f'$  is not continuous at  $c$ , then the above statement is not true. Consider the function  $f$  such that

$$f(x) = \begin{cases} x^2 \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}.$$

It can be shown that  $f'(0) = 0$  but  $\lim_{x \rightarrow 0} f'(x)$  does not exist because of the oscillating nature of  $f'$  near 0.

To further show that one cannot determine a derivative in this way, consider the function  $f$  such that

$$f(x) = \begin{cases} 0, & x \neq 0 \\ 1, & x = 0 \end{cases}.$$

We know that  $f$  is not differentiable at 0 since  $f$  is not continuous at 0, but  $\lim_{x \rightarrow 0} f'(x) = 0$ .

So if students believe that one can always find a derivative at a point  $c$  by taking the limit of the surrounding derivatives as you approach  $c$ , then this reveals that they do not understand the correct definition of the derivative.

I showed the class how they could use the built-in derivative feature on their graphing calculator to confirm their by-hand calculations. On the TI-86, the two first derivative operations are der1 and nDer. The second one returns an approximate numerical derivative of a function  $f$  for a given value of  $c$  by determining the slope of the secant line through the points  $(c - \delta, f(c - \delta))$  and  $(c + \delta, f(c + \delta))$ , where  $\delta$  is the value given in the tolerance window, which by default is 0.001. Entering nDer(|x|, x, 0) on the calculator home screen returns a value of 0, which is the calculated approximate numerical derivative of  $f(x) = |x|$  at  $c = 0$ . Actually, for any  $\delta$  the value will be 0, and further,

$$\lim_{\delta \rightarrow 0} \frac{|0 + \delta| - |0 - \delta|}{(0 + \delta) - (0 - \delta)} = \lim_{\delta \rightarrow 0} 0 = 0.$$

Of course, students should have already studied this function previously and seen that  $f$  is not differentiable at 0 by examining the one-sided derivatives. So here is a good opportunity to advise students to not take for granted what their calculator displays, and also to reemphasize the correct definition of the derivative.



For elementary functions, nDer usually returns for a given function  $f$  and number  $c$  a value very close to the actual derivative. This is because if  $f$  is differentiable at  $c$ , then

$$\begin{aligned} \lim_{\delta \rightarrow 0} \frac{f(c + \delta) - f(c - \delta)}{(c + \delta) - (c - \delta)} &= \\ \lim_{\delta \rightarrow 0} \frac{f(c + \delta) - f(c) + f(c) - f(c - \delta)}{2\delta} &= \\ \frac{1}{2} \left( \lim_{\delta \rightarrow 0} \frac{f(c + \delta) - f(c)}{\delta} + \right. \\ \left. \lim_{\delta \rightarrow 0} \frac{f(c) - f(c - \delta)}{-\delta} \right) &= \\ \frac{1}{2} (2f'(c)) &= f'(c). \end{aligned}$$

A positive result of actually defining the derivative of a function  $f$  at a number  $c$  in the interior of the domain of  $f$  by

$$f'(c) = \lim_{\delta \rightarrow 0} \frac{f(c + \delta) - f(c - \delta)}{2\delta}$$

is that for some functions that have corners and cusps, it gives a value for the derivative at these locations, as we saw in the case of  $f(x) = |x|$  at  $c = 0$ .

As another example, if

$$f(x) = \begin{cases} -4x, & x < 0 \\ x, & x \geq 0 \end{cases},$$

then it can be shown that  $f'(0) = -\frac{3}{2}$ .

However, considering this further, we have that

$$f'(x) = \begin{cases} -4, & x < 0 \\ -3/2, & x = 0 \\ 1, & x > 0 \end{cases}.$$

This implies that a derivative does not necessarily possess the intermediate value property.

Further, considering the function  $f$  given by

$$f(x) = \begin{cases} 0, & x \neq 0 \\ 1, & x = 0 \end{cases},$$

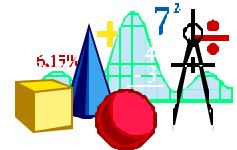
we would have that  $f'(x) = 0$  for all  $x$ . So the function  $f$  is differentiable at 0 even though it is not continuous at 0!

For the elementary functions students usually study in calculus, these various ways of looking at the derivative are equivalent, but their deeper implications are great and differing, so a precise and correct understanding of the standard accepted definition is paramount. Of course, if students don't have a solid grasp of the definition of the derivative, they can still know how to correctly use the basic differentiation rules, which is certainly one accomplishment. However, if we can get some of them to have a fuller and more accurate understanding of the derivative, then I believe we have truly succeeded in teaching them calculus.

David Stott ■

### Test Your Skills

As an extension of the article, try the following exercises.



- Show that for  $f(x) = x^2$  and  $c = 0$  that  $f'(0)$ ,  $\lim_{x \rightarrow 0} f'(x)$  and  $\lim_{\delta \rightarrow 0} \frac{f(0 + \delta) - f(0 - \delta)}{2\delta}$  all exist and are equal.
- Prove that for a given function  $f$  and number  $c$  in the interior of the domain of  $f$  that

$$f'(c) = \lim_{x \rightarrow c} f'(x) = \lim_{\delta \rightarrow 0} \frac{f(c + \delta) - f(c - \delta)}{2\delta}$$

iff  $f'$  is continuous at  $c$ .



3. Show that for

$$f(x) = \begin{cases} x \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

and  $c = 0$  that  $f'(0)$ ,  $\lim_{x \rightarrow 0} f'(x)$  and

$$\lim_{\delta \rightarrow 0} \frac{f(0 + \delta) - f(0 - \delta)}{2\delta}$$
 do not exist.

4. Show that for the function  $f(x) = x^{1/3}$  and  $c = 0$  that

$$f'(0) = \lim_{x \rightarrow 0} f'(x) = \lim_{\delta \rightarrow 0} \frac{f(0 + \delta) - f(0 - \delta)}{2\delta} = \infty .$$

5. Show that for

$$f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$$

and  $c = 0$  that

$$f'(0) = \lim_{\delta \rightarrow 0} \frac{f(0 + \delta) - f(0 - \delta)}{2\delta} = \infty ,$$

but that  $\lim_{x \rightarrow 0} f'(x) = 0$ .

6. Show that for

$$f(x) = \begin{cases} x^{2/3}, & x < 0 \\ x^2, & x \geq 0 \end{cases}$$

and  $c = 0$  that  $f'(0)$  and  $\lim_{x \rightarrow 0} f'(x)$  do not exist,

but that

$$\lim_{\delta \rightarrow 0} \frac{f(0 + \delta) - f(0 - \delta)}{2\delta} = -\infty .$$

7. Show that for  $f(x) = x^{2/3}$  and  $c = 0$  that  $f'(0)$  and  $\lim_{x \rightarrow 0} f'(x)$  do not exist, but that

$$\lim_{\delta \rightarrow 0} \frac{f(0 + \delta) - f(0 - \delta)}{2\delta} = 0 .$$

### Student Competition Awards



Several students received awards at the Spring Quarter Department Colloquium. Pictured are Oliver Newton (Problem of the Week Contest winner, WI/02), David Hocking (SP/02 AMATYC Competition – Third Place), Andy Lawler (2001–02 AMATYC Competition – Fifth Place) and Brett Holland (2001–02 AMATYC Competition – Third Place).



HwaPyong Ko received the second-place score on the Spring Quarter AMATYC Competition and tied with Linh Liu (not pictured) for the first-place cumulative score for the 2001–02 academic year. Each student received half of the \$100 cumulative first-place award.



(Continued from Page 1) Retention Program, making several hundred phone calls each quarter, meeting individually with about thirty students each quarter and offering 5-6 workshops and review sessions to about 100 students each quarter.

In addition to all this, in the Fall Quarter we completed a very enlightening student attendance study in Math 101. It was discussed in a very positive front-page article in the *Clarion* featuring the Mathematics Department, and has been the subject of much discussion and the impetus for further studies in other departments. Also, five faculty members traveled to professional conferences this year, Susan Myers was able to offer her Mathematical Modeling course for the first time since 1996, we underwent an external department evaluation by Dr. Thomas Gantner, recently retired chair of University of Dayton's Mathematics Department, and we awarded Marilyn Thompson the Part Time Faculty Member of the Year Award. (See page 2.) And in the last four quarters we have taught 10,628 students in 432 sections of mathematics courses. This amounts to 3134.67 FTE's, which is 8.65% of the college total.

Finally, I want to extend my sincerest congratulations to Sy Ostransky, Susan Myers and John Pftzing, who are all celebrating their retirements this year. Thank you to each one of you for your many years of dedicated service to our students, the department and the college. May you thoroughly enjoy your retirement in good health and in good cheer. We are grateful for all you have given.

And thank you also to each member of the full- and part-time staff for a busy and successful year. May you have a blessed and relaxing summer and come back refreshed and ready for another exciting year in the Fall.

Al Giambrone ■



**Tom Gantner, retired Math Department Chair from the University of Dayton, performed the department's external review this May, which is conducted once every four years. Pictured are Al Giambrone, Tom Gantner, Barb Carruth, David Stott, and Gwen English.**

**Harvey's Joke Corner**

Q. Why did the golfer wear two shirts?  
A. Because he had a hole in one.

A wife's concern about her husband's drinking:  
"He sees double and he thinks he's single."



The Math Department inventory for the past fiscal year noted that most of the tables had no legs.

Greeting for new teachers: "Welcome to 'show' business."

Harvey Chew ■