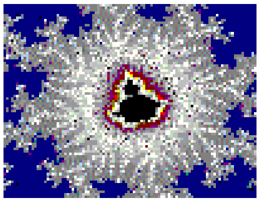

Sinclair Mathnet

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FROM THE CHAIR

In elementary calculus we make quite a point of teaching a variety of methods for deciding whether or not an infinite series converges. But as far as finding the sum of a convergent series, all that is covered is the formula for summing a geometric series and the technique for summing a telescoping series. In the past two weeks I have had the good fortune to discover more about summing convergent series than I had learned in the thirty-five years (am I really admitting this?) since I have studied calculus. It started with a problem that appeared on the AMATYC student mathematics competition exam that puzzled me at first: Find $\sum_{k=1}^{\infty} \frac{k}{8^k}$. Though this is not geometric, an approach similar to the technique for deriving the formula for a geometric series works.

Find $\sum_{k=1}^{\infty} \frac{k}{8^k}$.

Solution: Let $S_n = \frac{1}{8} + \frac{2}{8^2} + \cdots + \frac{n}{8^n}$.

$$\frac{1}{8} S_n = \frac{1}{8^2} + \frac{2}{8^3} + \cdots + \frac{n-1}{8^n} + \frac{n}{8^{n+1}}.$$

Subtracting gives

$$\frac{7}{8} S_n = \frac{1}{8} + \frac{1}{8^2} + \cdots + \frac{1}{8^n} - \frac{n}{8^{n+1}}$$

$$= \frac{\frac{1}{8} - \left(\frac{1}{8}\right)^{n+1}}{1 - \frac{1}{8}} - \frac{n}{8^{n+1}}.$$

$$S_n = \left(\frac{8}{7}\right)^2 \left[\frac{1}{8} - \left(\frac{1}{8}\right)^{n+1}\right] - \left(\frac{8}{7}\right)\frac{n}{8^{n+1}}.$$

Taking the limit yields

$$\sum_{k=1}^{\infty} \frac{k}{2^k} = \frac{8}{49}.$$

In Chapter 3 of William Dunham's book, *Euler -The Master of Us All* (the most recent undertaking of the Mathematics Department Book Club), I learned that Jakob Bernoulli used some slightly more sophisticated versions of this same approach to sum a variety of other series.

For example, consider $\sum_{k=1}^{\infty} \frac{k^2}{2^k}$.

We have $S_n = \frac{1}{2} + \frac{4}{4} + \frac{9}{8} + \cdots + \frac{n^2}{2^n}$ and

$$\frac{1}{2} S_n = \frac{1}{4} + \frac{4}{8} + \frac{9}{16} + \cdots + \frac{(n-1)^2}{2^n} + \frac{n^2}{2^{n+1}}.$$

Subtracting gives

$$\frac{1}{2} S_n = \frac{1}{2} + \frac{3}{4} + \frac{5}{8} + \cdots + \frac{n^2 - (n-1)^2}{2^n} - \frac{n^2}{2^{n+1}}. \quad (1)$$

The first n terms on the right amount to

$$\sum_{k=1}^{n} \frac{2k-1}{2^k} = 2\sum_{k=1}^{n} \frac{k}{2^k} - \sum_{k=1}^{n} \frac{1}{2^k}. \quad (2)$$

The first sum on the right of (2) is

$$2^2 \left(\frac{1}{2} - \left(\frac{1}{2}\right)^{n+1}\right) - (2)\frac{n}{2^{n+1}}$$

using the same method we used in the first problem.

The second sum on the right of (2) is $\dfrac{\frac{1}{2}-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}}$.

Substituting into (2) gives

$$\sum_{k=1}^{n}\frac{2k-1}{2^k}=8\left(\frac{1}{2}-\left(\frac{1}{2}\right)^{n+1}\right)-\frac{4n}{2^{n+1}}-\frac{\frac{1}{2}-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}}.$$

Then substituting into (1) gives

$$\frac{1}{2}S=8\left(\frac{1}{2}-\left(\frac{1}{2}\right)^{n+1}\right)-\frac{4n}{2^{n+1}}-\frac{\frac{1}{2}-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}}-\frac{n^2}{2^{n+1}}.$$

Multiplying by 2 and taking the limit yields

$$\sum_{k=1}^{\infty}\frac{k^2}{2^k}=16\left(\frac{1}{2}\right)-2(1)=6.$$

Though I haven't tried it, I assume that a similar but more complicated version of the above approach will show that $\sum_{k=1}^{\infty}\frac{k^3}{2^k}=26$.

When Euler enters the stage, however, the methods become far more ingenious, though perhaps somewhat lacking in rigor. To sum $\sum_{k=1}^{\infty}\frac{1}{k^2}$, he defines a judiciously chosen power series and treats it as a polynomial. He identifies its roots and uses the Factor Theorem to write it as an infinite product.

Using the factored form of the polynomial, he expresses the coefficient of the x^2 term as an infinite series that he sets equal to the coefficient of x^2 in the power series form. Multiplying this equation by $-\pi^2$ dramatically exposes the original series, $\sum_{k=1}^{\infty}\frac{1}{k^2}$, on one side and $\frac{\pi^2}{6}$ on the other!

In another more rigorous proof of the same result he separates the series into the sum of two series, one including the even-numbered terms and the other the odd-numbered terms. Factoring $\frac{1}{4}$ from the even-termed series reveals the series to be evaluated and reduces the problem to evaluating the odd-termed series. This he does by showing that a certain definite integral is equal to that odd-termed series when its integrand is expanded into a series and integrated. Then he calculates the same integral using the Fundamental Theorem of Calculus.

I encourage you to read *Euler - The Master of Us All* if you would like to learn more about summing infinite series and many other remarkably creative ideas of this brilliant mathematician.

Al Giambrone ■

In Appreciation of Euler…

If we compared the Bernoullis to the Bach family then Leonhard Euler (1707 – 1783) is unquestionably the Mozart of mathematics, a man whose immense output – not yet published in full – is estimated to fill at least seventy volumes. Euler left hardly an area of mathematics untouched, putting his mark on such diverse fields as analysis, number theory, mechanics and hydrodynamics, cartography, topology, and the theory of lunar motion. With the possible exception of Newton, Euler's name appears more often than any other throughout classical mathematics. Moreover we owe to Euler many of the mathematical symbols in use today, among them *i*, *π, e and f*(*x*). And as if that were not enough, he was a great popularizer of science, leaving volumes of correspondence on every aspect of science, philosophy, religion, and public affairs.

\- Eli Maor

(From his book *e - The Story of a Number)*

Fifth-Year Anniversary

This issue marks the fifth-year anniversary of *Mathnet*, as well as *Mathnet*'s 31st issue. The editors of *Mathnet* would like to thank all the part-time and full-time faculty and staff who have contributed to *Mathnet* during this time. These contributions are what have made and continue to make *Mathnet* a successful and valuable communication resource for the Department.

The editors would also like to especially thank the "regular" contributors: Al Giambrone, Harvey Chew, and John Pfetzing. Also, special appreciation is given to Sy Ostransky for many of the "Test Your Skills" problems.

Mathnet has seen a number of subtle changes since the first four-page, white issue, which was published in March of 1995. The most important development was in March 1997 when we began using the Xerox Docutech in Central Duplicating to obtain high-quality copies. This definitely gave us much better photographs, as can be seen by looking at the February 1997 issue!

Martin Moore, standing next to the Xerox Docutech in Central Duplicating, provides a critical link in the publication of *Mathnet*.

The editors of *Mathnet* would like to give special recognition to Martin Moore in Central Duplicating for his great cooperation, time, and effort in ensuring a successful printing of *Mathnet*.

Lastly, if anyone has seen the Department web page, they will see that back-issues of *Mathnet* may be found there. This is the work of Len Ruth, who has with great diligence converted each Word document to HTML format. In the past year this work has been helped by the use of pdf-converting software. A special thanks to Len for all his assistance in getting *Mathnet* on the web.

The editors believe that *Mathnet* is fulfilling its intended purpose, which, as expressed by Al in the first "From the Chair" article, is "to benefit the part-time faculty by keeping them better informed of the activities of the Department, and to give them a sense of more intimate involvement in the life of the Department." However, we also believe that all the part-time and full-time faculty and staff are receiving great benefit from *Mathnet*, and we hope to continue this worthwhile effort.

▪ Editors of *Mathnet*

REMINDERS

- Please remember to give your students a syllabus on the first day of the term that includes all the elements identified on pages 1.3 and 1.4 of our department handbook. Also make sure the department gets a copy of it. If you need a handbook, contact the office.
- Remember that department policy limits extra credit to at most 10% of the total points available for a student's grade and that typically it is much lower than that. See page 4.1 in the handbook for the full policy statement on this.
- Remember that all sections of Math 101 and 102 are to give a two-part final exam. The first part is supplied by the department and the second part by the instructor. Neither part is optional.

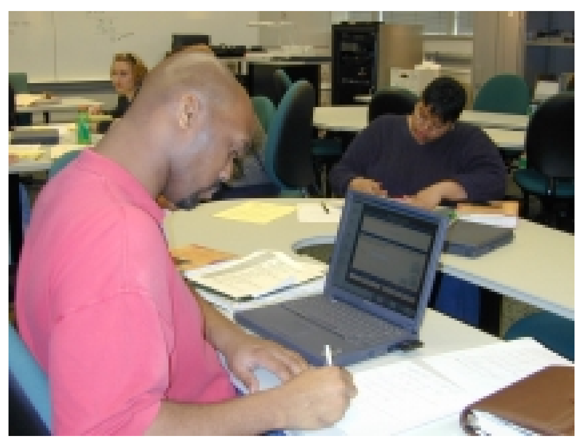
MATH 101 LEARNING CHALLENGE PROJECT

The Mathematics Department is currently involved in a pilot project to identify MAT 101 students who are at risk of failing in the traditional classroom, and to help as many as possible to succeed by offering them an alternative style of instruction. Two special sections of MAT 101 started off Winter Quarter as traditional sections taught by Kay Cornelius and Susan Harris. Students were informed about the program on the first day of classes and were encouraged to talk with their instructor about any concerns they had regarding the program.

After Test #1 (at the end of Week 2) these 2 sections were reconfigured into 2 "new" classes: one class was to continue to learn algebra using a traditional format with Kay, and the other class was to begin using alternative methods with Susan. Criteria used to form the new classes included scores on Test #1, background data collected on the first day of the quarter, a Learning Styles Inventory and a Study Skills Checklist. Generally, students who seemed to be at risk of failure in the traditional class were assigned to the alternative class. Because of equipment constraints, the traditional class has 34 students and the alternative, 29.

The alternative class is meeting in Room 14-306 (CIL), supplied with new furniture, 24 wireless computers, 5 PCs and a printer, all supplied by the CIL. Students there have immediate access to tutorial software ("Introductory Algebra" by Quant Systems), the TV-Sinclair MATH 101 Videos (which were digitized onto CDs), the usual traditional textbook, worksheets, activities, the tutorial assistance of 2 tutors and the instructor, and a few mini-lectures by Susan. Since these students had demonstrated some deficiencies on Test #1, each began the alternative class with a checklist of Unit 1 topics that were to be reviewed along with suggested methods of how to accomplish this. Homework was assigned, collected and graded. Students were given the opportunity to re-test over Unit 1 before proceeding to the new material. This same plan will be followed for the remaining Unit Tests: a checklist of topics, directions on how to use various learning modes, and homework collected and graded before attempting a Unit Test. The Final Exam will be taken only once. Tests are being taken in the Testing Center or during the 2 extra "open lab" hours per week.

The students are being permitted some flexibility in covering the material and taking tests, but Susan will try to keep as many students as possible on a schedule to finish by the end of the quarter. Two extra hours per week of open lab are being conducted in the classroom with Susan and 1 tutor, and students have access to the software at most other times in another lab area in Building 14. The students may even check out the software to take home. If appropriate, it will be possible for Susan to issue an "incomplete" to a student who is nearly finished with the course requirements, but needs up to an extra 3 weeks to complete the course.



The software being used has been provided by Quant Systems, free of charge for this pilot only. It has been installed on the CIL network, which is accessible only from the CIL. At the end of the first pilot, it will be uninstalled. The software is being used on a trial basis only; therefore, the project committee must decide whether or not to use it again for the second pilot (Fall, 2000).

Assessing the first pilot has already begun with a short questionnaire to assess students' perceptions of the course. Other assessment tools will include a more extensive questionnaire near the end of the quarter, an instructor questionnaire, and several statistical analyses comparing the experimental group (both special sections this quarter) to a control group according to grades, withdrawals, and success in the next math course. During the spring and summer quarters, the project committee will continue the assessment process, in addition to revising the course design for the second pilot. The feasibility of expanding the program on a permanent basis will also be studied.

■ Susan Myers

Retreat Team Update

The Retreat Team was very pleased with the success of the family picnic last summer. It was a beautiful day, and all participants had a great time with the team activities and nature hike, not to mention the great food!

The retreat team has now commenced looking forward to this summer's retreat, which will be a faculty retreat similar to the retreat at Bergamo Center held the summer before last. The retreat will include a variety of informative and entertaining components, as well as some team challenges and surprises. The Team's highest goal is to provide an opportunity whereby both part-time and full-time members of the Department can get to know one another better, as well as to allow for a greater sharing of ideas concerning our teaching, classroom experiences, and the future of mathematics education at Sinclair.

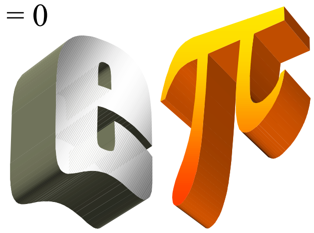
The Retreat Team is considering a more full retreat day this year in order to accommodate the many activities planned. The Team would like to have the retreat from morning until mid-afternoon or about 3:30. Included will be a breakfast, lunch and snack.

If any members of the Department have any questions, comments, or suggestions about the upcoming retreat, they may direct them to any member of the Retreat Team.

- Retreat Team
  Lyn Keeler, David Stott, Tom Wilson

Mathematical Patterns

Mathematics abounds with beautiful patterns and symmetry. Here is a list of some nice results concerning π and e.



Euler's formula: $e^{\pi i} + 1 = 0$

$i^i = e^{-\pi/2}$

$$2 = \frac{e^1}{e^{1/2}} \cdot \frac{e^{1/3}}{e^{1/4}} \cdot \frac{e^{1/5}}{e^{1/6}} \cdot \ldots$$

$\ln 2 = 1 - 1/2 + 1/3 - 1/4 + - \ldots$

$e = 1/0! + 1/1! + 1/2! + 1/3! + 1/4! + \ldots$

(Newton 1665)

$$e = 2 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{2}{3 + \cfrac{3}{4 + \cfrac{4}{5 + \ldots}}}}}$$

(Euler 1737)

$$\frac{2}{\pi} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \cdot \ldots$$

(Francois Viete 1593)

$$\frac{\pi}{2} = \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \frac{6}{7} \cdot \ldots$$

(John Wallis 1655)

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$$

(James Gregory 1671)

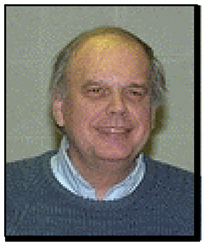
$$\frac{2}{\pi} = 1 - 5\left(\frac{1}{2}\right)^3 + 9\left(\frac{1 \cdot 3}{2 \cdot 4}\right)^3 - 13\left(\frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}\right)^3 + \cdots$$

(Srinivasa Ramanujan 1913)

$$\int_0^a e^{-x^2} dx = \tfrac{1}{2}\pi^{1/2} - \frac{e^{-a^2}}{2a+} \, \frac{1}{a+} \, \frac{2}{2a+} \, \frac{3}{a+} \, \frac{4}{2a+\cdots}$$

(Srinivasa Ramanujan 1913)

Just Thinking



- What is the speed of dark?
- When not in your right mind, does your left mind get pretty crowded?
- What happens when you get scared to death twice?
- Why is it that experience is something you don't get until just after you need it?
- Why is it no one is listening until you make a mistake?

There were three medieval kingdoms on the shores of a lake. There was an island in the middle of the lake, which the kingdoms had been fighting over for years. Finally, the three kings decided that they would send their knights out to do battle, and the winner would take the island.

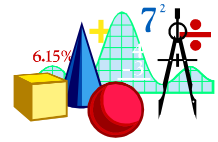
The night before the battle, the knights and their squires pitched camp and readied themselves for the fight. The first kingdom had twelve knights and each knight had five squires, all of whom were busily polishing armor, brushing horses, and cooking food. The second kingdom had twenty knights, and each knight had ten squires. Everyone at that camp was also busy preparing for battle. At the camp of the third kingdom, there was only one knight with his squire. This squire took a large pot and hung it from a looped rope in a tall tree. He busied himself preparing the meal while the knight polished his own armor.

When the hour of battle came, the three kingdoms sent their squires out to fight (this was too trivial a matter for the knights to join in). The battle raged, and when the dust cleared, the only person left standing was the lone squire from the third kingdom, having defeated the squires from the other two kingdoms.

Thus it was proved that the squire of the high pot and noose is equal to the sum of the squires of the other two sides.

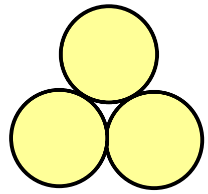
John Pfetzing ■

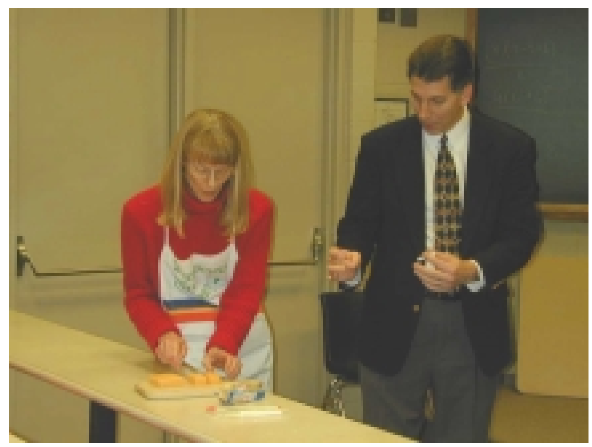
Test Your Skills



We hope you have time to investigate this problem, and to offer your solution to either Lyn Keeler or David Stott.

What is the area of the gap between these circles of radius 1?





Vicki Lair "cuts the cheese" during Dr. Mark Oxley's presentation *Arrangements of Hyperplanes* at the Winter Quarter Department Colloquium.

Harvey's Joke Corner

Math instructor's favorite pain relievers:
Trig instructors: "Sine-aid"
Algebra instructors: "Equate"

Advertising for purchasing life insurance for your cat:
Contact the Nine Lives Life Insurance Company - buy insurance for 1/9 of the human price!

A couple met at a blood pressure screening - they found they had a common denominator. They married when he was 50 and she was 40, but broke up a year later. They no longer complemented each other.

Harvey Chew ■