

Interest Formulas from Business and Finance

Many of which are exponential in form

1) Definition of Quantities

P : Amount initially borrowed or deposited

A : Total amount gained or owed

r : Annual interest rate

r_{eff} : Effective annual interest rate

t : Time period in years for an investment

T : Time period in years for a loan

N : # of compounding periods per year

M : Monthly payment.

$$e : e = \lim_{n \rightarrow \infty} \left[1 + \frac{1}{n} \right]^n$$

2) Simple Interest

1. Interest alone: $I = PrT$

2. Total repayment over T :

$$R = P + PrT =$$

$$P(1 + rT)$$

3. Monthly Payment over T :

$$M = \frac{P(1 + rT)}{12T}$$

3) Simple Principle Growth and Decline

1. Compounded Growth: $A = P\left(1 + \frac{r}{N}\right)^{Nt}$

2. Continuous Growth: $A = Pe^{rt}$

3. Annual Inflation Rate r : $A = Pe^{-rt}$

4) Effective Interest Rates

1. N Compounding Periods per year:

$$r_{eff} = \left(1 + \frac{r}{N}\right)^N - 1$$

2. Continuous Interest: $r_{eff} = e^r - 1$

3. Known P, A, T : $r_{eff} = \sqrt[T]{\frac{A}{P}} - 1$

5) Continuous Interest IRA Growth Formulas

1. Annual Deposit D Evenly Metered

$$\text{Throughout the Year: } A = \frac{D}{r}(e^{rt} - 1)$$

2. Annual Deposit D (evenly metered)

Plus Initial Deposit P :

$$A = Pe^{rt} + \frac{D}{r}(e^{rt} - 1)$$

3. Replacement Formula:

Continuous Interest to
Compounded Interest

$$\text{Replace } e^{rt} \text{ with } \left(1 + \frac{r}{N}\right)^{Nt}$$

6) Continuous Interest Mortgage Formulas

1. First Month's Interest: $I_{1st} = \frac{rP}{12}$

2. Monthly Payment: $M = \frac{Pr e^{rT}}{12(e^{rT} - 1)}$

3. Total Repayment ($P + I$): $A = \frac{PrTe^{rT}}{e^{rT} - 1}$

4. Total Interest Repayment:

$$I = P \left[\frac{rTe^{rT}}{e^{rT} - 1} - 1 \right]$$

5. Replacement Formula:

Continuous Principle Reduction to
Monthly Principle Reduction

$$\text{Replace } e^{rT} \text{ with } \left(1 + \frac{r}{12}\right)^{12T}$$