

3.3 Measures of Central Tendency: Means, Medians, Modes

Measures of Center

1. Mean:

The mean is the arithmetic average. The sample mean of $\{x_1, x_2, \dots, x_N\}$ is given

by $\bar{x} = \frac{x_1 + x_2 + \dots + x_N}{N}$ and the population mean is given by

$$m = \frac{x_1 + x_2 + \dots + x_N}{N}$$

2. Median:

The median is the value closest to the middle in a sorted data set.

To find the median

- Put the data in order of size
- Find the data value in the middle. If there is no unique middle value, then average the two values closest to the middle. [The median is in the

$$\frac{N+1}{2} \text{ position}]$$

3. Mode

The number that occurs most frequently is the mode. There can be more than one mode at a time. If all the data values are found equally often then there is no mode.

Example: Find the sample mean, median and mode of the following ages: $\{17, 38, 21, 108, 51, 44, 24, 30, 27\}$.

$$\text{Sample mean is } \bar{x} = \frac{17 + 38 + 21 + 108 + 51 + 44 + 24 + 30 + 27}{9} = \frac{360}{9} = 40.$$

To find the median, first sort the data: 17, 21, 24, 27, 30, 38, 44, 51, 108. The middle number is in the $\frac{9+1}{2} = \frac{10}{2} = 5^{\text{th}}$ position, which is 30.

Since every number appears once there is no mode

Note: The mean can be affected by abnormally large or small data values, but the median is not affected. In this example the age of 108 made the sample mean much larger than the corresponding median. The median would be the same regardless of the exact value of the largest age.

Example: Find the population mean, median and mode of the following quiz grades: $\{10, 8, 3, 7, 9, 8, 5, 6\}$.

$$\text{Population mean is } m = \frac{10 + 8 + 3 + 7 + 9 + 8 + 5 + 6}{8} = \frac{56}{8} = 7.$$

To find the median, first sort the data: 3, 5, 6, 7, 8, 8, 9, 10. The middle number is in the $\frac{8+1}{2} = \frac{9}{2} = 4.5^{\text{th}}$ position, which is the average of the number in the

4th position (7) and the number in the 5th position (8), which is $\frac{7+8}{2} = \frac{15}{2} = 7.5$.

The mode is 8 since it appears twice.

4. Weighted Means

If the data values x_1, x_2, \dots, x_N have weights of w_1, w_2, \dots, w_N then the weighted

mean is $\frac{w_1x_1 + w_2x_2 + \dots + w_Nx_N}{w_1 + w_2 + \dots + w_N}$

Example: Find your GPA if you had an
A in Math 101 (4 credit hours)
B in Math 102 (5 credit hours)
A in Math 108 (3 credit hours)

Here the weights would be the number of credit hours, and the data would be the numerical equivalent of the grade (A=4, B=3, etc) so with

$w_1 = 4, w_2 = 5, w_3 = 3, x_1 = 4, x_2 = 3, x_3 = 4$ we have a weighted average of

$$\frac{4(4) + 5(3) + 3(4)}{4 + 5 + 3} = \frac{43}{12} = 3.583$$

The notes above are for Math 108, Math for the Modern World using *Mathematics in Life, Society and the World 2nd edition* by Parks, Musser, Burton, and Siebler. Prentice Hall 2000.